THE UNIVERSITY OF ALBERTA

EXTENSIONS AND REFINEMENTS
OF THE METHODS AND CONCEPTS IN
THE STRUCTURE OF ATONAL MUSIC

by

© TOM GULAS

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ABSTRACT

The purpose of this study is to examine the applications and theoretical foundations of an analytic method proposed by Forte in The Structure of Atonal Music. Pitch is the only parameter employed in this method which categorizes pitch into twelve pitch-classes. Contextual and abstract musical examples are described by pitch-classes in unordered structures. For this reason, the method has been referred to as a set theoretic approach because the pitch-classes are represented by integers, and their containment in unordered segments is analogous to elementary set theory.

A great deal of attention has been given to the set theoretic approach in general, and Forte's text in particular. The conclusions arrived at throughout this study indicate that this attention should be redirected because of theoretical inconsistencies and discrepencies in the application of Forte's method. The intention of this thesis will not be to propose a replacement theory, but to identify weaknesses and inadequacies in Forte's theories, now in common use.

The material in Chapters 2-6, inclusive, is concerned with an examination of various theoretical concepts developed by Forte. In most cases, improvements or refinements of these concepts are described.

The final chapter, which deals with application, consists for the most part of alternative analyses of pieces that Forte discusses. Although dealing almost exclusively with pitch, the alternative analyses are not based on any preconceived theory or approach. However, the set theoretic method is employed in representing musical pitches by integers. After each analysis, a summary of Forte's analysis is presented in order to evaluate his analytic method. The final section summarizes the main conclusions of these evaluations plus statements of a general nature.

Although Forte's nomenclature of pitch-class sets has become accepted as a reference for continuing research, his theory and application has not gained general acceptance. This thesis will show that this attitude is justified.

PREFACE

Much of the material presented by Forte in The Structure of Atonal Music is a synthesis of previous research by Forte and other authors. The general approach that Forte employs has been called a set theoretic method. This concept is an extension of theory dealing with the pitch parameter in serial music. Since this theory has been widely disseminated, knowledge of concepts such as integer notation, pitch-classes, transposition and inversion within modulo 12, and complementation will be assumed.

Although not a strict requirement for the understanding of this study, familiarity with Forte's text would be beneficial.

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CHAPTER 1

INTRODUCTION AND BACKGROUND

This chapter is concerned with a presentation and brief review of the literature devoted to atonal analysis and theory. Following this summary, sections of Forte's text[1] and the attention they have received will be discussed. It is within this context that this thesis is justified.

The writings of Babbitt are fundamental in the field of serial music, but are also responsible for the foundations of atonal theory. Much of the mathematical formalism and the introduction of the concept of pitch-classes (pcs, pc's, PC's) with their representation by integers (modulo 12) are due to Babbitt.[2] This numerical approach has been called a set theoretic method because segments of notes are depicted by sets (segments) of integers (pcs). With few exceptions, the set theoretic approach has been continuously applied to atonal analysis and theory, even though it was originally for use in serial theory.

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1 Allen Forte, <u>The Structure of Atonal Music</u> (New Haven and London: Yale University Press, 1973).

2 Milton Babbitt, "Set Structure as a Compositional Determinant," <u>Journal of Music Theory</u>, V/2 (1961), 49-79, and "Twelve Tone Invariants as Compositional Determinants," <u>Musical Quarterly</u>, XLVI (1960), 246-259.

A notable exception is Perle's text[3] which is an accepted reference source in the field of atonal and serial analysis. Due to the novelty of the set theoretic method at the time of the first edition, its omission by Perle is quite understandable. A more isolated exception is Browne, who says

> "My point is that I don't think that pitch class (octave) equivalence holds primacy of place among the ways we do (let alone must) recognize relatedness among pitches and pitch structures." [4]

In terms of theory, it is advantageous to disregard register in the discussion of pitch relations. However in analysis, it is at the discretion of the analyst whether or not register is included (pitch) or excluded (pitch-class). Early examples employing integer notation are present in conjunction with conventional notations of pitch, e.g., staff and alphabetic name.[5]

The next major development in atonal theory is "A Theory of Set-Complexes for Music."[6] This article introduces the concepts of interval-class similarity and the unification of inclusion and complementation within the set complex principle. The concept of complementation had been described earlier.[7]

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3 George Perle, Serial Composition and Atonality (Berkeley: University of California Press, 1962).

4 Richmond Browne, "Review of The Structure of Atonal Music," Journal of Music Theory, XVIII/2 (1974), p. 396.

5 Allen Forte, "Context and Continuity in an Atonal Work: A Set Theoretic Approach," Perspectives of New Music, I/2 (1963), 72-82.

6 idem, Journal of Music Theory, VIII/2 (1964), 136-183.

7 Milton Babbitt, "Set Structure," and "Twelve Tone Invariants," plus David Lewin, "The Intervallic Content of a

Comments of a general nature that apply to this article are contained in reviews.[8] This article also had a direct influence on another study which contains analyses of serial and atonal examples.[9] Forte's article contains the analysis of only one atonal piece. Over the next few years, the field of atonal theory and analysis received little attention in the published literature. Relevant studies that did appear concentrated on theory[10] or analysis.[11]

The most significant achievement in the field is The Structure of Atonal Music. This text ressembles Forte's earlier article on set complexes. It contains a few revisions in terminology, e.g., the earlier list of prime forms is rearranged and interval-class similarity relations are labelled R_0, R_1, or R_2. Other concepts are introduced for the first time, e.g., bips (basic interval patterns)[12] and R_p (an inclusion relation).

Collection of Notes," Journal of Music Theory, IV/1 (1960), 98-101, and "Intervallic Relations between Two Collections of Notes," Journal of Music Theory, III/2 (1959), 298-301.

8 John Clough, "Pitch-Set Equivalence and Inclusion (A Comment on Forte's Theory of Set-Complexes)," Journal of Music Theory, IX/1 (1965), 163-171. Allen Forte, "The domain and Relations of Set-Complex Theory," Journal of Music Theory, IX/1 (1965), 173-180.

9 Richard Teitelbaum, "Intervallic Relations in Atonal Music," Journal of Music Theory, IX/1 (1965), 72-127.

10 Miriam Goder, "The Interval-Triangle," Journal of Music Theory, XVI (1972), 142-167.

11 Allen Forte, "Sets and Nonsets in Schoenberg's Atonal Music," Perspectives of New Music, XI/1 (1972), 43-64.

12 idem, "The Basic Interval Patterns," Journal of Music Theory, XVII/2 (1973), 234,272.

Browne reviews the text but includes only general observations.[13] Benjamin gives a more detailed and valuable discussion of the concepts and methodology employed by Forte.[14] The majority of subsequent works do not have the balance of theory and analysis that are present in Forte's text. These studies are directed toward theoretical[15] or analytical[16] considerations. This is a general observation because the theoretical works often contain brief examples of analysis, and the analytical studies usually include a discussion of methodology which can not be referred to as a presentation of a theory. Beach's article is an exception.[17] Although his analyses are not based on Forte's theories, he does include a thorough, but brief, summary of Forte's methodology

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13 Browne, "Review of The Structure of Atonal Music," 390-415.

14 William E. Benjamin, "The Structure of Atonal Music by Allen Forte," Perspectives of New Music, XIII/1 (1974), 170-190.

15 Alan Chapman, "Some Intervallic Aspects of Pitch-Class Set Relations," Journal of Music Theory, XXV/2 (1981), 275-290. Charles H. Lord, "Intervallic Similarity Relations in Atonal Set Analysis," Journal of Music Theory, XXV/1 (1981), 91-111. Robert Morris, "A Similarity Index for Pitch-Class Sets," Perspectives of New Music, XVIII/2 (1980), 445-460. Daniel Starr, "Sets, Invariance and Partitions," Journal of Music Theory, XXII/1 (1978), 1-42.

16 William E. Benjamin, "Ideas of Order in Motivic Music," Music Theory Spectrum, I (1979), 23-34. Allen Forte, "Aspects of Rhythm in Webern's Atonal Music," Music Theory Spectrum, II (1980), 90-109, The Harmonic Organization of "The Rite of Spring" (New Haven and London: Yale University Press, 1978). Christopher Hasty, "Segmentation and Process in Post Tonal-Music," Music Theory Spectrum, III (1981), 54-73.

17 David W. Beach, "Pitch Structure and the Analytic

In terms of theory or analysis, The Structure of Atonal Music has not been thoroughly examined and evaluated, yet it continues to be a standard reference. In spite of the status of Forte's text, surprisingly, only Appendix 1 which lists pc sets is universally accepted in the field. Forte's theories and analytical method have largely been neglected. Even though Forte's nomenclature is generally accepted as a standard, it has been necessary in the following chapters to adopt different terms and nomenclature. Pc sets are referred to as n-groups, and instead of describing them by interval vectors, (n-1)-group contents are employed. Since n-groups are numerically ordered according to their (n-1)-group contents, their order and nomenclature differ from Forte's pc sets. It is anticipated that the results obtained employing (n-1)-group contents (unification of (R_0, R_1, R_2,and R_p) will justify the inconvenience of a new nomenclature.

With respect to similarity relations, the literature contains a concept that unifies Forte's three relations involving interval vectors (R_0, R_1, and R_2).[18] This thesis develops a concept that unifies these concepts, plus R_p, within a general principle of subgroup inclusion.

Process in Atonal Music: An Interpretation of the Theory of Sets," Music Theory Spectrum, I (1979), 7-22.

18 Lord, "Intervallic Similarity Relations," and Morris, "A Similarity Index."

Bips are briefly discussed in the thesis, and only a few suggestions to extend the concept are provided. Although worthy of greater attention,[19] Forte's text does not devote a lot of discussion to the bip's theoretical implications or analytical applications.

Pairs of distinct pc sets that have identical interval vectors have been referred to as Z-pairs since Forte's early article on the set complex. The property of Z-pairs (but not the term) is first described by Lewin, who writes that "I am unable to find any explanation for the exceptional behavior of the pentachords of (4) and (5)."[20] He is referring to Z-pairs of cardinal number five. It should be mentioned that there are three such pairs, signifying that Lewin has overlooked one such pair. To the present, no adequate explanation of Z-pairs has appeared. This thesis includes, for the first time, a discussion elucidating the Z-pair property.

The set complex has not been dealt with in the literature to a great extent. Forte employs the set complex as the basis of his analytical method. This study will concentrate not on its practical applications (analysis), but on its theoretical implications.

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19 Forte, "The Basic Interval Patterns."
20 Lewin, "The Intervallic Content," p. 100.

The final chapter of this study is concerned with analysis and is not dependent on material in the preceding chapters, which deal primarily with theoretical aspects. To appraise the analytical section of Forte's text, a comparative approach is employed. After a detailed analysis of a piece is presented, a summary of Forte's analysis (of the same piece) is given. Finally, the two analyses are compared so that Forte's analytical method can be evaluated. The alternative analyses are not intended to exemplify a theory, although they are dependent on the set theoretic method. Therefore, the analyses do not incorporate Forte's theoretical concepts nor the suggested refinements contained in the preceding chapters. These analyses are included not to demonstrate theoretical principles, but to evaluate an analytical approach.

The conclusions arrived at throughout the thesis indicate deficiencies and weaknesses in most of Forte's theoretical concepts. (This may explain their absence in current research). The proposed suggestions for their extension are intended to encourage further investigation in the field of theory. The final chapter concludes that Forte's application of his theory to analysis, which employs only the set complex, is heavily biased in favour of obtaining predetermined results. As such, Forte's application of the set theoretic approach within his own theoretical concepts has not been pursued by other authors.

Although the following chapters are critical of _The Structure of Atonal Music_, the thesis is not intended to downgrade the esteemed position of Forte's text. As a pioneering study, it contains original insights and contributions in the fields of theory and analysis. The degree of originality in the text has not been surpassed, in either field, since its publication. The most valuable contribution of _The Structure of Atonal Music_ has been the stimulus that it has provided for encouraging research in the fields of atonal analysis and theory.

CHAPTER 2

DESCRIPTION AND NOMENCLATURE OF GROUPS

The difference between a pc (pitch-class) set and a group is that the latter can contain repetition of pcs, but the former, as defined by Forte, can not. A prefix (n or an arabic numeral) to "group" is an indication of the number of different pcs in the group, which is referred to as the cardinal number. A prefixed group contains no repetition of pcs and is, therefore, identical to a pc set. Due to the possibility of repeated pcs in a musical context, the number of pcs in a group may be different from the cardinal number of that group.

In the appendix (Table XI) n-groups of cardinal number two through twelve are listed with their pcs in integer notation. The ordering of n-groups within each cardinal number is based on a descending succession of their (n-1)-group, or subgroup, content.

A subgroup is a group which has a smaller cardinal number than the group under discussion. An (n-1)-group is a subgroup whose cardinal number is one less than the n-group being referred to. (n-2), (n-3), etc., groups are also subgroups.

The (n-1)-group content is a list of the (n-1)-groups contained within an n-group. For example, 3:9 has an (n-1)-group content of 2:2 5^{2}, which signifies that 3:9 contains one 2:2 and two 2:5's. In the notation of n-groups, a colon separates the ordinal from the cardinal number to differentiate this system from Forte's, which employs a hyphen.

The list of prime forms in Forte's appendix orders pc sets according to their 2-group (interval-class) vectors. A vector is a notational device which indicates, with six digits, the quantity of each 2-group. 3:9 has a 2-group vector of 010020.

This method of ordering has two faults. Firstly, there are pairs of n-groups that possess the same 2-group vector. The n-groups (within each pair) do not follow one another in the list despite their identical 2-group vectors. Instead, only one member of the pair is included in the ordering by 2-group vectors, while the remaining n-group is placed at the end of the list. Referred to as Z-related pairs (which are discussed in Chapter 5), these pairs are an inconsistency in the ordering procedure.

Secondly, the ordering and description of n-groups according to their 2-group vectors is not a consistent approach. As an example, 4-groups are described by (n-2)-group vectors, 5-groups by (n-3)-group vectors, and so on. As the complexity (cardinal number) of n-groups increases, the measure of ordering should also increase, and yet be relative in relation to all n-groups. Ordering n-groups by

their 2-group vectors is not a relative descriptive practice.

Ordering n-groups by their (n-1)-group contents is consistent for each n-group, and avoids Z-related pairs because every (n-1)-group content is associated with only one n-group. (n-1)-group contents are listed in Table XI, instead of (n-1)-group vectors, because they are more manageable in terms of required space for their presentation. (n-1)-group vectors are unwieldy in this respect, e.g., 6-group vectors have fifty digits. Since subgroup contents and vectors indicate the same information, the ordering and listing of all n-groups necessitated the use of (n-1)-group contents. Vector notation is, nevertheless, useful in other applications which are dealt with in Chapter 3.

Total invariance is the property of an n-group which enables its pcs to be invariant after a transformation has occurred. These transformations are inversion or transposition. An n-group that is not invariant by either of these operations possesses twenty-four distinct forms (including itself): twelve transpositions and twelve transpositions of the inversion. N-groups that are invariant by inversion or transposition will be referred to as IBI or IBT groups, respectively. IBI or IBT groups have less than twenty-four different forms. Table XI lists only one form of each n-group.

The (n-1)-group content can be used to determine whether or not an n-group is IBI, IBT, or both. In most cases only a summary of the (n-1)-group content is required. The (n-1)-group distribution provides this information by indicating the quantity of each different (n-1)-group contained in an n-group. It can be easily derived from the (n-1)-group content. 3:$\underline{9}$ has an (n-1)-group distribution of 1 2, and 3:[$\underline{12}$] has one of 3. An n-group that has an (n-1)-group distribution of singletons, e.g., 1 1 1, or 1 1 1 1 1, is non-IBI and non-IBT.

N-groups that are IBI are indicated by a "_" beneath their ordinal number, e.g., 3:$\underline{9}$. IBI groups have (n-1)-group contents with double occurrences of (n-1)-groups. When these double entries are related by inversion, and single entries are associated with IBI (n-1)-groups, the n-group which contains them is IBI. Figure 1 exemplifies the previous statements about IBI groups.

	pcs	(n-1)-group content	(n-1)-group distribution
IBI group 5:$\underline{5}$	(0,2,3,4,6)	4:2^2 8^2 $\underline{22}$	1 2 2

(n-1)-groups 4:2 (0,2,3,4) — Inversion
4:2 (2,3,4,6)
4:8 (0,2,3, 6) — Inversion
4:8 (0, 3,4,6)
4:$\underline{22}$ (0,2, 4,6) — IBI

Figure 1. Relations between (n-1)-groups in an IBI group

Not all double occurrences in the (n-1)-group contents are related by inversion. There are a few exceptions which are contained within non-IBI groups (Figure 2). In these cases, the pairs are related by transposition. 6:[47] is the only IBT group that is not also IBI. 5:32/7:27, 4:15/8:23, and 5:24/7:35 are complementary with respect to 12:[$\underline{1}$].

```
         5:32  (0,1,4,5,8)              6:35  (0,1,3,4,6,9)
               (0,1,4,  8)  4:15              (0,1,3,  6,9)  5:24
               (0,  4,5,8)  t=6               (0,  3,4,6,9)  t=3

         (0,1,2,4,5,6,8,9)  8:23              (0,1,3,4,6,7,9)  7:35
   7:27  (0,1,2,4,5,  8,9)              6:35  (0,1,3,4,6,  9)
   t=4   (0,1,  4,5,6,8,9)              t=3   (0,  3,4,6,7,9)

                     6:[47]  (0,1,3,6,7,9)
                             (0,1,3,6,  9)  5:24
                             (0,  3,6,7,9)  t=6
                             (  1,3,6,7,9)  5:25
                             (0,1,3,  7,9)  t=6
                             (0,1,3,6,7  )  5:31
                             (0,1,  6,7,9)  t=6
```

Figure 2. Relation between two (n-1)-groups in non-IBI n-groups

Although two occurrences of an (n-1)-group within an IBI n-group have been seen to be related by inversion (Figure 1), this results from the fact that the (n-1)-groups are non-IBI. The relation is different if the (n-1)-groups are IBI (Figure 3).

4:$\underline{22}$ (0,2,4,6)

3:$\underline{6}$ (0,2,4)
3:$\overline{6}$ (2,4,6) —IBI: related by transposition
3:$\overline{8}$ (0,2, 6)
3:8 (0, 4,6) —non-IBI: related by inversion

Figure 3. Relation between two IBI and two non-IBI (n-1)-groups within an IBI n-group

To conclude, IBI n-groups have double occurrences of (n-1)-groups which can be determined from the (n-1)-group distribution. If the (n-1)-group is non-IBI, the two appearances are related by inversion, but if it is IBI, its two occurrences are related by transposition (Figure 3). Single occurrences of an (n-1)-group in an IBI n-group are always IBI (Figure 1). N-groups that are IBI only (not also IBT) have twelve forms which are related by transposition to the form listed in Table XI.

N-groups that are IBT are indicated by a "[ ]" around their ordinal number, e.g., 6:[47]. The number of transpositions which create total invariance of IBT groups can be determined from their (n-1)-group contents. Table I lists all IBT groups, their (n-1)-group content, and the number of transpositions that create invariance, which will be referred to as the degree of IBT. For example 9:[<u>12</u>] is IBT of degree 3. The degree of invariance is equal to the quantity of each IBI (n-1)-group (8:<u>25</u> in 9:[<u>12</u>]), or one-half the number of each non-IBI (n-1)-group (8:23 in 9:[<u>12</u>]). 6:[47] is the only exception to this rule, and it is the only IBT n-group that is not also IBI, as noted previously (Figure 2).

TABLE I

IBT Groups and their Degrees of Invariance

IBT Group	(n-1)-group Content	Levels of Invariance
$2{:}[\underline{6}]$		2
$3{:}[\underline{12}]$	$2{:}\underline{4}^{3}$	3
$4{:}[\underline{21}]$	$3{:}5^{4}$	2
$4{:}[\underline{28}]$	$3{:}8^{4}$	2
$4{:}[\underline{29}]$	$3{:}\underline{10}^{4}$	4
$6{:}[\underline{32}]$	$5{:}\underline{15}^{2}\ 16^{4}$	2
$6{:}[47]$	$5{:}24^{2}\ 25^{2}\ 31^{2}$	2
$6{:}[\underline{49}]$	$5{:}32^{6}$	3
$6{:}[\underline{50}]$	$5{:}\underline{36}^{6}$	6
$8{:}[\underline{22}]$	$7{:}18^{4}\ 19^{4}$	2
$8{:}[\underline{27}]$	$7{:}\underline{24}^{2}\ 25^{4}\ \underline{33}^{2}$	2
$8{:}[\underline{29}]$	$7{:}35^{8}$	4
$9{:}[\underline{12}]$	$8{:}23^{6}\ \underline{25}^{3}$	3
$10{:}[\underline{6}]$	$9{:}\underline{7}^{2}\ 8^{4}\ 9^{4}$	2
$12{:}[\underline{1}]$	$11{:}\underline{1}^{12}$	12

IBT n-groups of degree k have 24/k different types. 6[47] thus has twelve forms; six transpositions and six transpositions of its inversion. Since all the other IBT n-groups are also IBI, the invariance due to inversion halves the number of different forms to 12/k. 6:[$\underline{50}$] (whole tone scale) is IBT of degree 6 and has only two (12/6) types.

Less than total invariance occurs when there are less than n invariant pcs after some transformation of an n-group. In order that an n-group have only (n-1) invariant pcs, a non-IBI, or non-IBT, (n-1)-group must be present more than once in the n-group. If the n-group contains one IBI, or one IBT, (n-1)-group, then (n-1) pcs will be invariant depending on the operation (inversion or transposition) which causes invariance within the (n-1)-group itself.

Figure 42 on page 38 of Forte's text is a list of pc sets that hold (n-1) subsets invariant by transposition. However, no figure is provided for the case of inversion. This figure is inaccurate because it omits one example (Figure 4) and incorrectly lists 7-31 twice. In addition, the relevant discussion fails to provide an adequate understanding of the concept or property of subset inclusion and invariance. As such, it is one of several instances in the body of the text wherein significant detail is inadequately clarified.

```
7-28  (0,1,3,5,6,7,9)
t=6   (0,1,3,  6,7,9,11)

      (0,1,3,  6,7,9)  6-30:  invariant (n-1) subset
```

Figure 4. Transpositional invariance of 6-30 within 7-28

To this point, (n-1)-groups have provided a consistent description and ordering procedure for n-groups. This method has overcome faults in Forte's approach which employs interval vectors. The (n-1)-group content is capable of determining the type of invariance of n-groups, and thus the number of distinct forms. Forte does indicate the number of of different forms (enclosed in parentheses beside the prime forms in the appendix) of invariant pc sets, but no mention of the type of invariance involved.

In the following chapter, subgroups will be employed to extend, refine, and unify several concepts that Forte refers to as similarity relations. The importance of (n-1)-groups extends beyond their application to ordering and describing n-groups.

CHAPTER 3

SIMILARITY RELATIONS

Forte describes several similarity relations between groups of the same cardinal number. One of these, R_p, is based on common (n-1)-groups, and the other three, R_0, R_1, and R_2, are concerned with 2-group vectors.

Two or more n-groups are R_p related if they have in common at least one (n-1)-group. Is the presence of at least one (n-1)-group a sufficient test of similarity? The following example will demonstrate a weakness in R_p.

4:3, 4:4, and 4:20 have (n-1)-group contents of 3:$\underline{1}$ 3 4 7, 3:$\underline{1}$ 4 5 8, and 3:4 5 8 9, respectively. These three groups share various 3-groups (Figure 5). R_p is not capable of describing the number of 3-groups in common, but only the existence of at least one 3-group. Since 4:4 and 4:20 have three 3-groups in common, the similarity between them is greater than the similarity between 4:3 and 4:20, which have only one common 3-group.

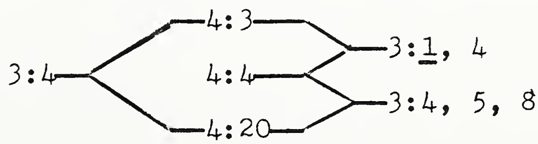

Figure 5. Mutual 3-groups in three 4-groups that are R_p related

This weakness in R_p can be overcome by indexing the degree of similarity, and directly indicating the number of mutual (n-1)-groups. 4:3 and 4:20 are R_p of degree 1, for example. To continue, 4:3 and 4:4 are R_p of degree 2, and 4:4 and 4:20 are R_p of degree 3.

An R_p-tuple is a list of n-groups that contain the same (n-1)-group. With the aid of Table XI, it can be verified that the following is an R_p-nontuple (9) because the nine 4-groups contain 3:4 - 4:3, 4, 9, 10, <u>13</u>, 15, <u>18</u>, 19, and 20.

If (n-1)-group vectors are available, the determination of R_p-tuples is much easier than the method employing (n-1)-group contents. The (n-1)-group vector of a 4-group contains twelve entries; there are twelve 3-groups. Listing twenty-nine 3-group vectors (there are twenty-nine 4-groups) results in a matrix with twelve columns and twenty-nine rows. An R_p-tuple can be determined by reading down any column (3-group) and noting those rows (4-groups) that have a non-zero entry. Despite this advantage of subgroup vectors over subgroup contents, the appendix contains (n-1)-group contents because of their facility in presentation, as discussed in Chapter 2.

R_0, R_1, and R_2 are similarity relations dealing with 2-group vectors. Two n-groups that have an unequal number of each 2-group are R_0 related (Figure 6a — identical to Example 50, page 49, of Forte's text). This relation is termed minimum similarity. R_1 and R_2 are called maximum similarity because n-groups in either of these relations have equal numbers of four 2-groups (Figure 6b and 6c — drawn from page 48 of Forte's text).

			interchange		no interchange
4-2	221100	4-2	221100	5-10	223111
4-13	112011	4-3	212100	5-Z12	222121
a. R_0		b. R_1		c. R_2	

Figure 6. Examples of R_0, R_1, and R_2 relations

R_0, R_1, and R_2 have at least one major flaw as similarity relations; they are incomplete. These relations account for cases in which two n-groups have identical quantities of zero (R_0) and four (R_1 and R_2) 2-groups. To be complete, these relations should be extended to include the situation in which identical numbers of one, two, or three 2-groups are present. These are the remaining cases because Z-related pairs have identical 2-group vectors (six 2-groups) and it is impossible to have only five types of 2-groups equally contained in two n-groups.

There is a more difficult problem in these relations that can not be corrected by mere extension. The following argument depends on contrived examples, but should elucidate the problem and suggest a manner of correction.

Consider pairs of students taking courses in six subject areas. Figure 7 illustrates the status of the students' progress in a manner analogous to 2-group vectors.

students	subject areas					
A	0	3	0	0	0	0
B	0	0	3	0	0	0
C	3	4	3	4	3	4
D	4	3	4	3	4	3
A'	0	21	0	0	0	0
B'	0	0	21	0	0	0

Figure 7. Course status of pairs of students

In the first example, the progress of students A and B is R_1 related: maximum similarity with interchange. This results from the absence of any courses in subject areas 1, 4, 5, and 6 by each student. Since the courses that each student participated in are not mutually shared, the students, in fact, have no courses in common. Is this maximum similarity?

In the case of students C and D, the situation is different. Using the criterion of similarity employed by Forte results in the conclusion that the status of these students is R_0 related: minimum similarity because they have not taken exactly the same number of courses in any subject area. As a foundation for similarity relations, this criterion is misleading because each student has completed three courses in each subject area: eighteen common courses. When students C and D have eighteen courses in common, in a program of twenty-one, is the term "minimum similarity" justified?

The final example concerns students A' and B' (A and B, after more courses have been completed). Each student completes twenty-one courses which are distinct in subject area from their partner's status. Their programs are still R_1 similar, but can they be regarded as maximally similar?

It should be apparent from the previous discussion that the criterion in evaluating R_0, R_1, and R_2 (exact correspondence of 2-groups) is not a sufficient foundation for similarity relations.

A more meaningful criterion would be to evaluate the number of 2-groups that are common to two n-groups. This is identical to the indexing of R_p described at the beginning of this chapter. Rather than dealing only with (n-1)-groups, R_p can be extended to include lower order subgroups. When the lower limit of 2-groups is reached, the indexing of the R_p principle is an improvement over R_0, R_1, and R_2. Table II indicates the lower limit of subgroups (2-groups) in relation to each n-group. With the same principle of similarity being applied to several subgroups (2, 3, 4, and 5-groups in relation to 6-groups), another nomenclature is required to replace R_p, which deals only with (n-1)-groups. The following system is suggested because it indicates the order of subgroup, and incorporates the indexing feature formerly applied to R_p. R_p is replaced by the term "(n-1) similar" with the degree of similarity following. As an example (Figure 5), 4:4 and 4:20 are (n-1) similar of degree 3. This describes directly that 4:4 and 4:20 have three 3-groups in common. This general concept will be referred to as (n-k) similarity, where k is variable.

TABLE II

Lower Limit of Subgroups in Relation to N-groups

N-groups	Lower Limit of Subgroups (2-groups)
3-groups	(n-1)-groups
4-groups	(n-2)-groups
5-groups	(n-3)-groups
6-groups	(n-4)-groups
7-groups	(n-5)-groups
8-groups	(n-6)-groups
9-groups	(n-7)-groups
10-groups	(n-8)-groups

The appendix (Table XI) lists only (n-1)-group contents and the appendix in Forte's text gives 2-group vectors. There are, therefore, a number of n-groups for which there is not a complete list of their total subgroup content, i.e., from (n-1)-groups (Table XI) and other n-groups whose cardinal number decreases from (n-1) to two (Forte's appendix). Such a list can be obtained by using the (n-1)-group contents directly, or the equivalent information in vector form

The (n-1)-group contents can be utilized to determine the (n-2)-group content of any n-group. By extension, this method can be applied to the calculation of any subgroup content, regardless of size. As an example, consider 4:2 and its (n-1)-group content of $3{:}\underline{1}\ 2\ 3\ \underline{6}$. A representation of the 2-group content of 4:2 can be inferred from the 2-group content of each 3-group (Figure 8). The final result is a representation of the 2-group content because each 2-group is included in two different 3-groups and is, therefore, double-listed. The correct 2-group content can be calculated by halving the number of each 2-group obtained by this method. Performing this operation yields a 2-group content (of 4:2) of $2{:}\underline{1}^2\ \underline{2}^2\ \underline{3}\ \underline{4}$.

* 4:2 – (n-1)-group content is $3{:}\underline{1}\ 2\ 3\ \underline{6}$ *

$3{:}\underline{1}$ – (n-1)-group content is $2{:}\underline{1}^2\ \underline{2}$
3:2 – (n-1)-group content is $2{:}\underline{1}\ \underline{2}\ \underline{3}$
3:3 – (n-1)-group content is $2{:}\underline{1}\ \underline{3}\ \underline{4}$
$3{:}\underline{6}$ – (n-1)-group content is $2{:}\underline{2}^2\ \underline{4}$

$2{:}\underline{1}^4\ \underline{2}^4\ \underline{3}^2\ \underline{4}^2$

* 4:2 – (n-2)-group content is $2{:}\underline{1}^2\ \underline{2}^2\ \underline{3}\ \underline{4}$ *

Figure 8. Calculation
of the 2-group content of 4:2

The layout of Figure 8 can accommodate the (n-2) group content of any n-group, not just for 4-groups. (n-3)-groups can be determined by taking the (n-1)-group content of each (n-2)-group, and calculating and summing as in Figure 8. This result must be divided by three to give the correct number of (n-3)-groups in an n-group. In fact, each time an (n-k)-group is calculated, a division of its quantity must take place. These divisions take on increasing values beginning with two for (n-2)-groups, three for (n-3)-groups, and so on, leading to the result that in the calculation of an (n-k)-group, a division of k is required. Figure 9 calculates the (n-4)-group content, by steps, indicating the necessary divisions.

* $6{:}[\underline{50}]$ – (n-1)-group content is $5{:}\underline{36}^6$ *

$5{:}\underline{36}^6$ – (n-1)-group content is $4{:}(\underline{22}^2\ \underline{24}^2\ [\underline{28}])^6$

$4{:}\underline{22}^{12}\ \underline{24}^{12}\ [\underline{28}]^6$

* $6{:}[\underline{50}]$ – (n-2)-group content is $4{:}\underline{22}^6\ \underline{24}^6\ [\underline{28}]^3$ *

$4{:}\underline{22}^6$ – (n-1)-group content is $3{:}(\underline{6}^2\ 8^2)^6$

$4{:}\underline{24}^6$ – (n-1)-group content is $3{:}(\underline{6}\ 8^2\ [\underline{12}])^6$

$4{:}[\underline{28}]^3$ – (n-1)-group content is $3{:}(8^4)^3$

$3{:}\underline{6}^{18}\ 8^{36}\ [\underline{12}]^6$

* $6{:}[\underline{50}]$ – (n-3)-group content is $3{:}\underline{6}^6\ 8^{12}\ [\underline{12}]^2$ *

$3{:}\underline{6}^6$ – (n-1)-group content is $2{:}(\underline{2}^2\ \underline{4})^6$

$3{:}8^{12}$ – (n-1)-group content is $2{:}(\underline{2}\ \underline{4}\ [\underline{6}])^{12}$

$3{:}[\underline{12}]^2$ – (n-1)-group content is $2{:}(\underline{4}^3)^2$

$2{:}\underline{2}^{24}\ \underline{4}^{24}\ [\underline{6}]^{12}$

* $6{:}[\underline{50}]$ – (n-4)-group content is $2{:}\underline{2}^6\ \underline{4}^6\ [\underline{6}]^3$ *

Figure 9. Calculation of the 2-group content of $6{:}[\underline{50}]$

Since subgroup vectors display the same information as subgroup contents, Figure 8 can be translated to illustrate the use of (n-1)-group vectors (Figure 10). As in Figure 8, the final result must be halved to produce the correct 2-group vector of (2 2 1 1 0 0). The numbers in round parentheses are matrices, even though two of them are only one row of numbers, and "X" denotes matrix multiplication.

$$\begin{array}{cc} & \begin{array}{cccc} 3{:}\underline{1} & 2 & 3 & \underline{6} \end{array} \\ 4{:}2 & \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} \end{array} \times \begin{array}{cc} & \begin{array}{cccccc} 2{:}\underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & [\underline{6}] \end{array} \\ \begin{array}{c} 3{:}\underline{1} \\ 3{:}\overline{2} \\ 3{:}3 \\ 3{:}\underline{6} \end{array} & \begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cccccc} 2{:}\underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & [\underline{6}] \end{array} \\ \begin{pmatrix} 4 & 4 & 2 & 2 & 0 & 0 \end{pmatrix} \end{array}$$

Figure 10. Preliminary calculation of the 2-group vector of 4:2

To this point, the subgroup vectors, or subgroup contents have been calculated for only one n-group. Access to a computer and a program that can manipulate matrices can provide the means by which the subgroup vecors can be determined for all n-groups.

Earlier in this chapter, it was mentioned that by converting (n-1)-group contents to (n-1)-group vectors, the result would be (n-1)-group matrices, i.e., matrices whose rows correspond to (n-1)-group vectors for each n-group. Multiplying these (n-1)-group matrices and dividing by an appropriate quantity will result in a matrix whose rows can be any desired subgroup vector. Figure 11 is a diagrammatic representation of the method to obtain the 3-group vectors for all 7-groups. The arabic numerals indicate which (n-1)-group matrix (obtained from Table XI) is represented. The subscripts show the number of rows and columns, respectively, in the matrix. Since 3-group vectors are (n-4)-group vectors (with respect to 7-groups), the matrix on the right side of the equation (A) must be divided by twenty-four (1 X 2 X 3 X 4) to obtain a matrix whose rows are 3-group vectors of 7-groups.

$$^{7}{}_{35\ 50} \times {}^{6}{}_{50\ 35} \times {}^{5}{}_{35\ 29} \times {}^{4}{}_{29\ 12} = {}^{A}{}_{35\ 12}$$

Figure 11. Diagrammatic representation of the calculation to obtain the 3-group matrix of 7-groups

This terminology (matrices, vectors) has been suggested as an extension, and not a replacement, of the general (n-k) similarity relation derived earlier. Admittedly, the concepts of matrices and their manipulation are peripheral to the pertinent discussion of similarity relations. Nevertheless, it is a procedure by which Table XI may be expanded to include any subgroup information that is required.

The purpose of this chapter has been to examine similarity relations. In the pursuit of this goal, it was determined that Forte's similarity relations were incomplete, inconsistent, and deficient. This necessitated a refinement in the form of (n-k) similarity, which is an extension of R_p and a unification of all Forte's similarity relations within one principle.

CHAPTER 4

ORDER RELATIONS

The previous chapters have treated groups as unordered entities, that is to say that the order of pcs has not been considered. A necessity for the inclusion of order relations would arise most naturally within a melodic or linear context, as opposed to a harmonic or vertical situation. The bip (basic interval pattern) is introduced by Forte to describe order relations.

Bips are a representation of the number and types of ics (interval-classes) in an ordered context. An example of bips, derived from ordered groups and their 2-group (ic) succession, is quoted from page 63 of Forte's text (Figure 12). The interval succession lists the 2-groups in the order that they occur in context, and the bip is merely a summary of these 2-groups arranged in increasing order.

5-4:	ordered set	interval succession	bip
A:	8 4 3 2 5	4-1-1-3	1134
B:	10 1 9 8 7	3-4-1-1	1134
C:	0 3 11 10 9	3-4-1-1	1134

Figure 12. Derivation of bips

Forte's discussion and application of bips to specific examples is confusing when it is confronted with repeated pcs. For example, Forte writes "Specifically, if a repetition occurs at order position m, then a new segment begins at (m-1)."[1] He applies this definition to the segment in Figure 13. This sole condition, however, seems inadequate. There is no reason why a segment can not also begin at order position (m-2) or pc6. In Figure 13, why would Forte not allow a segment to begin at order position (m-3) or pc7? Forte should have said that a segment must not contain a pc repetition, which is also the condition that he imposes on on a pc set itself.

order position m: repetition of pc4

4 8 4 3 2 5

4 6 8 4 3 2 5

4 7 6 8 4 3 2 5

Figure 13. Segmentation
when repeated pcs are present

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1 Allen Forte, <u>The Structure of Atonal Music</u> (New Haven: Yale University Press, 1973), p. 72.

The exception to the above rule, according to Forte, involves the situation in which the initial and final pcs of a segment are identical. If the segments contain n pcs, the segmentation produces two segments with (n-1) pcs each. Forte's example, on page 72 of the text, is unclear because the two examples actually overlap in context, and they are presented in reverse order with respect to their contextual appearance. These examples should have been combined into one. Figure 14 accomplishes this and indicates Forte's segmentation and order of presentation. What is most unexpected in this example, is that it fails to include preceding and subsequent material. This omission is not conducive to establishing the bip as a concept that is dependent on context.

this segment is shown second

8 7 1 10 8 2 11 1

this segment is shown first

Figure 14. Combination
of two segments discussed by Forte

The bip, as a description of 2-groups, can still be applied to segments with repeated pcs, if the succession of 2-groups is, indeed, a significant concept. For example, the bip 1134 represents a 5-group because it contains four conjunct 2-groups. The same bip also describes the 2-group succession in the following groups: (5,6,5,8,4) which is a 4-group – (4,5,6,8), and (5,6,5,2,6) which is a 3-group – (2,5,6). If the bip is to be a significant concept, it should not be restricted in use to segments that have no pc repetition. Furthermore, the bip should also be extended to include the succession of conjunct n-groups of higher cardinal number than two, though these are not as aurally perceptible as 2-group successions.

Forte does not include the bip within other concepts, nor is it employed in his analyses. For this reason, the discussion of bips has been brief, and contains remarks dealing with Forte's presentation and only a few suggestions concerned with extensions of the bip concept.

CHAPTER 5

Z-PAIRS AND COMPLEMENTATION

Forte uses the term "Z" to designate a relation between two n-groups that have identical 2-group vectors. This property was initially described by Lewin.[1] In addition, he also discussed a relation between the 2-group vectors of complementary n-groups (with respect to 12:[1]), which has been referred to as the complement theorem.[2] Complementary groups are two or more groups that have no pcs in common, but all their pcs form a referential n-group, usually 12:[1]. The complement theorem when applied to 6-groups, states that complementary 6-groups have identical 2-group vectors. In the event that one 6-group is not equal to its complement by transposition, or inversion followed by transposition, the complementary pair is Z-related.

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1 David Lewin, "The Intervallic Content of a Collection of Notes," *Journal of Music Theory*, IV/1 (1960), 98-101.
2 Daniel Starr, "Sets, Invariance and Partitions," *Journal of Music Theory*, XXII/1 (1978), 1-42.

Thus, the complement theorem is capable of explaining the existence of Z-pairs of cardinal six. This chapter will show that the remaining Z-pairs can be explained by complementation with respect to n-groups other than 12:[1]. Before this is undertaken, the 2-group square will be introduced and utilized as a method of illustrating the complement theorem as it applies to 12:[1]. Then the 2-group square will be applied to other n-groups so that non-6-group Z-pairs can be explained.

A 2-group square is a table that indicates the 2-groups formed by the pcs of an n-group with themselves. Table III is the 2-group square of 12:[1]. Pcs are in the same order in the columns and rows, which results in a symmetrical figure, accounting for the diagonal of 0's. Although the numerical value of pcs is increasing, this choice is arbitrary because any ordering could have been employed. Because of this symmetry, a 2-group formed by two pcs is present twice in the 2-group square. As an example, 2:5 is formed by pc2 and pc7, so a 5 is found at the intersection of the eighth row (pc7) and third column (pc2), and at the intersection of the third row (pc2) and eighth column (pc7). Both of these entries are symmetrical about the zero diagonal. This means that the actual 2-group content of 12:[1] is equal to one-half of the total number of 2-groups in the 2-group square.

TABLE III

2-group Square of 12:[$\underline{1}$]

Pcs	Pcs												2-group Summary
	0	1	2	3	4	5	6	7	8	9	10	11	
0	0	1	2	3	4	5	6	5	4	3	2	1	222221
1	1	0	1	2	3	4	5	6	5	4	3	2	222221
2	2	1	0	1	2	3	4	5	6	5	4	3	222221
3	3	2	1	0	1	2	3	4	5	6	5	4	222221
4	4	3	2	1	0	1	2	3	4	5	6	5	222221
5	5	4	3	2	1	0	1	2	3	4	5	6	222221
6	6	5	4	3	2	1	0	1	2	3	4	5	222221
7	5	6	5	4	3	2	1	0	1	2	3	4	222221
8	4	5	6	5	4	3	2	1	0	1	2	3	222221
9	3	4	5	6	5	4	3	2	1	0	1	2	222221
10	2	3	4	5	6	5	4	3	2	1	0	1	222221
11	1	2	3	4	5	6	5	4	3	2	1	0	222221

From the 2-group square directly, or the summary of the 2-groups in each row of Table III (each of the six digits is a tally of the number of each 2-group in the row), it can be determined that the 2-group square contains twenty-four entries, each, of 2:$\underline{1}$, 2:$\underline{2}$, 2:$\underline{3}$, 2:$\underline{4}$, and 2:$\underline{5}$, but only twelve 2:[$\underline{6}$]'s. The 2-group content of 12:[$\underline{1}$] is, therefore, $2{:}\underline{1}^{12}\ \underline{2}^{12}\ \underline{3}^{12}\ \underline{4}^{12}\ \underline{5}^{12}\ [\underline{6}]^{6}$.

Figure 15 is a representation of a 2-group square for an (m+n)-group that has been partitioned into two complementary groups: A_m and B_n, with cardinal numbers m and n, respectively. The cardinal number of this unspecified, referential group is, therefore, m plus n. Area X indicates the location of 2-groups that are formed by the pcs in A_m, and Y contains 2-groups formed by the pcs in B_n. The 2-groups in Z are formed by one pc from A_m and the other pc from B_n. From the discussion of the 2-group content of 12:[1], it is evident that the 2-group content of A_m and B_n is one-half the number of 2-groups in X and Y, respectively.

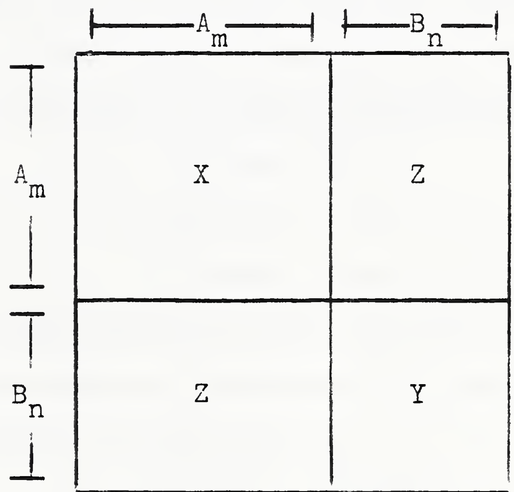

Figure 15. Generalized 2-group square of an (m+n)-group

If A_m and B_n are complementary with respect to 12:[1], then m plus n must equal twelve. The pcs of 12:[1] can be arranged so that its 2-group square is not unlike Figure 15. With this accomplished, the 2-groups in areas X, Y, and Z can illustrate the complement theorem.

Let D be the difference between the 2-group vectors of A_m and B_n, and assume that m is greater or equal to n. From the 2-group summary of each row (Table III), it can be seen that each row has the same number of each 2-group, i.e., one 2:[6] and two each of the remaining 2-groups. The rows of the 2-group square formed by pcs in A_m occupy the upper m rows, and the lower n rows are aligned with the pcs in B_n. The m rows contain m 2:[6]'s and 2m entries, each, of 2:1, 2:2, 2:3, 2:4, and 2:5. Similarly, the n rows contain n 2:[6]'s and 2n entries of the remaining 2-groups. The 2-groups in the upper m rows are contained in areas X and Z, and the 2-groups in the lower n rows are associated with areas Z and Y. The 2-group content of A_m and B_n is one-half the number of 2-groups in X and Y, respectively. Therefore:

$$\begin{aligned} D &= (1/2)(X - Y) \\ &= (1/2)(X + Z - Z - Y) \\ &= (1/2)(X + Z) - (1/2)(Z + Y) \end{aligned}$$

The number of each 2-group in (X + Z) and (Z + Y), as stated earlier, is dependent on the values of m and n, respectively. The value of D for 2:[6] is:

$$\begin{aligned} D &= (1/2)(m) - (1/2)(n) \\ &= (1/2)(m - n) \end{aligned}$$

For the remaining 2-groups, D has the value of:

$$\begin{aligned} D &= (1/2)(2m) - (1/2)(2n) \\ &= m - n \end{aligned}$$

The two values of D (depending on the 2-groups) describe the complement theorem. These differences apply to all values of m and n, whose sum is twelve, and when m is greater or equal to six. When m and n are equal, A_m and B_n represent complementary 6-groups. In this case, (m - n) and D are equal to zero, which signifies that complementary 6-groups have identical 2-group vectors. When one 6-group and its complement are not the same type of 6-group, they are Z-related and constitute a Z-pair.

The complement theorem, when applied to 12:[$\underline{1}$], simplifies the task of explaining the Z-pairs of cardinal number four and eight (1), and five and seven (3), because they are complementary within 12:[$\underline{1}$]. Therefore, it is only necessary to explain the Z-pair of cardinal number four and the three Z-pairs of cardinal number five.

An extension or variation of the complement theorem can be obtained from the 2-group square of an n-group if each row contains the same 2-group distribution. The 2-group square of 12:[$\underline{1}$] has this characteristic because it contains one 11-group twelve times. Although there is only one 11-group (11:$\underline{1}$), the foregoing statement is not trivial. Any n-group that contains one (n-1)-group n times has a 2-group square whose rows contain the same number and types of 2-groups. In other words, any n-group with an (n-1)-group distribution of n has this property.

Table IV lists all n-groups that have an (n-1)-group distribution of n, as can be verified with Table XI. Since the Z-pair of cardinal number four is complementary to some 8-group, it seems appropriate to examine the 2-group square of 8:[29].

TABLE IV

N-groups with an (n-1)-group Distribution of N

N-group	(n-1)-group
12:[1]	11:1
8:[29]	7:35
6:[49]	5:32
6:[50]	5:36
4:[21]	3:5
4:[28]	3:8
4:[29]	3:10
3:[12]	2:4

Table V contains the 2-group square of 8:[29] and a summary of the 2-group entries in each row. The extension of the complement theorem for 8:[29] states that the difference between the 2-group vectors of two complementary groups (A_m and B_n) is (1/2)(m - n) for all 2-groups except 2:3, in which case the difference is (m - n). Complementary 4-groups, within 8:[29], have equal 2-group vectors because (m - n) has the value zero.

TABLE V

2-group Square of 8:[29]

Pcs	0	1	3	4	6	7	9	10	2-group Summary
0	0	1	3	4	6	5	3	2	112111
1	1	0	2	3	5	6	4	3	112111
3	3	2	0	1	3	4	6	5	112111
4	4	3	1	0	2	3	5	6	112111
6	6	5	3	2	0	1	3	4	112111
7	5	6	4	3	1	0	2	3	112111
9	3	4	6	5	3	2	0	1	112111
10	2	3	5	6	4	3	1	0	112111

Of the thirty-five complementary 4-group pairs, all but eight pairs contain the same type of 4-group. The eight exceptions are pairs of 4:12 and 4:16: the Z-pair of cardinal number four. Not only are 4:12 and 4:16 complementary within 8:[29], but 8:[29] is the only 8-group that can contain a disjunct combination of the pcs in a 4:12 and 4:16. An extension of the complement theorem has, therefore, explained the Z-pair of cardinal number four as arising from complementation with respect to 8:[29].

From Table IV, it is clear that the complement theorem can not be extended and applied to a 10-group in order to explain the Z-pairs of cardinal number five. Nevertheless, just as the Z-pair of cardinal number four is complementary to only one 8-group, the Z-pairs of cardinal number five (each pair separately) are complementary to only one 10-group. One Z-pair of cardinal number five is complementary to 10:2, whose 2-group square is included in Table VI, along with a summary of the 2-group entries in each row.

TABLE VI

2-group Square of 10:2

Pcs	Pcs 0	1	2	3	4	5	6	7	8	10	2-group Summary
0	0	1	2	3	4	5	6	5	4	2	121221
1	1	0	1	2	3	4	5	6	5	3	212121
2	2	1	0	1	2	3	4	5	6	4	221211
3	3	2	1	0	1	2	3	4	5	5	222120
4	4	3	2	1	0	1	2	3	4	6	222201
5	5	4	3	2	1	0	1	2	3	5	222120
6	6	5	4	3	2	1	0	1	2	4	221211
7	5	6	5	4	3	2	1	0	1	3	212121
8	4	5	6	5	4	3	2	1	0	2	121221
10	2	3	4	5	6	5	4	3	2	0	022221

Although not all the rows have identical 2-groups, a pattern exists (Figure 16a). The requirement for a 10-group to have complementary 5-groups with identical 2-group vectors is that the total number of 2-group entries in five rows be identical to the other five rows. The pcs associated with the rows in each of these partitions form complementary 5-groups. Row types E and F differ in the number of 2:1's and 2:5's, which is also the source of difference between row types A and C. Combining these row types within two partitions with equal 2-group entries produces a 3-group (3:[12]) that must be contained in any 5-group of a complementary pair with identical 2-group vectors (Figure 16b). The two remaining pcs (of the complementary 5-groups) must be chosen such that each 5-group contains a pc from both row types; there are only four possibilities (Figure 16c). Combining these pairs of pcs with the two 3:[12]'s produces four pairs of complementary 5-groups that have identical 2-group vectors (Figure 16d). 5:11 and 5:19 are, therefore, a Z-pair. These two 5-groups are complementary to 10:2, which is the only 10-group that can contain a disjunct combination of 5:11 and 5:19. The following discussion will verify the previous statement. With respect to 12:[1], the complement of 5:11 is 7:20, which contains only one 5:19. Therefore, these two 5-groups can be complementary to only one 10-group. Alternatively, the 12-group complement of 5:19 is 7:26, which contains no more than one 5:11.

row type	2-group summary	pcs associated with each row type
A	121221	0-8
B	212121	1-7
C	221211	2-6
D	222120	3-5
E	222201	4
F	022221	10

a. Summary of row types

row type	pc	2-group summary	row type	pc	2-group summary
A	0	121221	C	2	221211
A	8	121221	C	6	221211
E	4	222201	F	10	022221
2-group total		464643	2-group total		464643
pc total	(0,4,8)=3:[12]		pc total	(2,6,10)=3:[12]	

b. Equalization of 2-groups in row types A, C, E, and F

row type	B D	row type	B D
pc pairs	1-3	pc pairs	7-5
	1-5		7-3
	7-3		1-5
	7-5		1-3

c. Pairs of pcs belonging to row types B and D

pcs	5-groups	5-groups	pcs
(0,1,3,4,8)	5:19	5:11	(2,5,6,7,10)
(0,1,4,5,8)	5:32	5:32	(2,3,6,7,10)
(0,3,4,7,8)	5:32	5:32	(1,2,5,6,10)
(0,4,5,7,8)	5:19	5:11	(1,2,3,6,10)

d. 5-group pairs with identical 2-group vectors

Figure 16. Complementary 5-groups with equal 2-group vectors

The two remaining Z-pairs of cardinal number five are both complementary within 10:[6]. Table VII contains the 2-group square of 10:[6] and a 2-group summary of each row.

TABLE VII

2-group Square of 10:[6]

Pcs	0	1	2	3	4	6	7	8	9	10	2-group Summary
0	0	1	2	3	4	6	5	4	3	2	122211
1	1	0	1	2	3	5	6	5	4	3	212121
2	2	1	0	1	2	4	5	6	5	4	220221
3	3	2	1	0	1	3	4	5	6	5	212121
4	4	3	2	1	0	2	3	4	5	6	122211
6	6	5	4	3	2	0	1	2	3	4	122211
7	5	6	5	4	3	1	0	1	2	3	212121
8	4	5	6	5	4	2	1	0	1	2	220221
9	3	4	5	6	5	3	2	1	0	1	212121
10	2	3	4	5	6	4	3	2	1	0	122211

Proceeding, as with 10:2, the row types can be identified (Figure 17a). Complementary 5-groups with equal 2-group vectors have one pc from a row type C and two pcs, each, from row types A and B. Z-pairs account for eight out of the thirty-five 5-groups that have equal 2-group vectors and are complementary within 10:[6] (Figure 17b). The remaining twenty-seven pairs of 5-groups contain two forms of the same 5-group.

row type	2-group summary	pcs associated with each row type
A	122211	0-4-6-10
B	212121	1-3-7-9
C	220221	2-8

a. Summary of row types

pcs and rows			5-groups	5-groups	pcs and rows		
A	B	C			A	B	C
4-6	1-7	2	5:26	5:10	0-10	3-9	8
0-10	1-7	8	5:26	5:10	4-6	3-9	2
0-10	3-9	2	5:26	5:10	4-6	1-7	8
4-6	3-9	8	5:26	5:10	0-10	1-7	2
4-10	3-7	2	5:13	5:23	0-6	1-9	8
4-10	1-9	8	5:13	5:23	0-6	3-7	2
0-6	3-7	8	5:13	5:23	4-10	1-9	2
0-6	1-0	2	5:13	5:23	4-10	3-7	8

b. 5-group Z-pairs that are complementary within 10:[6]

Figure 17. List of complementary 5-groups, within 10:[6], that are Z-related

This chapter has demonstrated, with the use of the 2-group square, that Z-pairs and complementation are related. This explanation of Z-pairs suggests that complementation, with respect to a referential n-group, need not be confined in its application to 12:[1]. Table VIII is a list of the Z-pairs of cardinal number four and five and an illustration of another similarity within these pairs.

TABLE VIII

Common Subgroup Segmentation within Z-pairs

Z-pair	N-group Containing the Z-pair	Complement of the N-group	Pcs of the Z-pair	Remaining Group
4:12 4:16	8:[29]	4:[29] (two 2:3's)	0 1 3 7 4 6 9 10	4:[29] (two 2:[6]'s)
5:11 5:19	10:2	2:2	2 5 6 7 10 0 1 3 4 8	3:[12]
5:10 5:26	10:[6]	2:[6]	8 9 10 0 3 1 2 4 6 7	3:6
5:13 5:23	10:[6]	2:[6]	2 3 4 7 10 6 8 9 0 1	3:4

CHAPTER 6

THE SET COMPLEX

The set complex and its extension, the subcomplex, are concepts based on the principles of complementation and subgroup inclusion. Since these concepts have already been discussed in Chapters 1, 2, and 4, it will be only their interaction within the set complex relation that is now examined.

Of all the concepts that Forte develops, the set complex is the most logical and complete, yet, it is also the most abstract and, hence, the most challenging. Some of these difficulties in comprehension arise from Forte's presentation. The definition of the set complex (K) is: $S/\overline{S} \in K(T,\overline{T})$ iff $S \supset\!\subset T/S \supset\!\subset \overline{T}$, which, in verbal terms, means that a set S or its complement ($\overline{S}$) is a member of the set complex about T or its complement if and only if S can contain or be contained in T, or S can contain or be contained in $\overline{T}$. The subcomplex (Kh) is an extension of K and is defined as: $S/\overline{S} \in Kh(T,\overline{T})$ iff $S \supset\!\subset T \& S \supset\!\subset \overline{T}$. The symbol "$\supset\!\subset$" is used to accommodate the proper relation between the different cardinal numbers of S and T, which are notated as #(S) and #(T), respectively.

With reference to the definition of Kh, Forte states that "From the previous discussion it should be evident that this definition produces 4 relations for any S and T that satisfy it because $S \subset T \& S \subset \overline{T}$ iff $\overline{T} \subset \overline{S} \& T \subset \overline{S}$."[1] Indeed, there are four relations, but not because of the single relation he provides. The four relations actually depend on #(S) and #(T) (Figure 18) and can not all be derived from Forte's single relation. Forte may be correct in stating that only one relation is necessary, however, #(S) and #(T) must be chosen such that $\#(T) > \#(S) < \#(\overline{T})$, where "$<$" and "$>$" mean less than and greater than, respectively. Since S or $\overline{S}$ and T or $\overline{T}$ are interchangeable, the choice of #(S) and #(T) is arbitrary. While only one choice satisfies Forte's single relation, he fails to mention this restriction and states, instead, that the relation holds true for "any S and T." This weakness should not reflect on the principles of K and Kh, which were explained, with greater clarity, by Forte in an earlier study.[2]

$S \subset T \& S \subset \overline{T}$ iff $\overline{T} \subset \overline{S} \& T \subset \overline{S}$, and when $\#(T) > \#(S) < \#(\overline{T})$
$S \subset T \& S \supset \overline{T}$ iff $\overline{T} \subset \overline{S} \& T \supset \overline{S}$, and when $\#(T) > \#(S) > \#(T)$
$S \supset T \& S \subset \overline{T}$ iff $\overline{T} \supset \overline{S} \& T \subset \overline{S}$, and when $\#(T) < \#(S) < \#(\overline{T})$
$S \supset T \& S \supset \overline{T}$ iff $\overline{T} \supset \overline{S} \& T \supset \overline{S}$, and when $\#(T) < \#(S) > \#(\overline{T})$

Figure 18. The four relations in Kh and the permissible values of #(S) and #(T)

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1 Allen Forte, The Structure of Atonal Music (New Haven: Yale University Press, 1973), p. 96.

2 Idem, "A Theory of Set-Complexes for Music," Journal of Music Theory, VIII/2 (1964), pp. 136-183.

Forte's presentation of general aspects of the subcomplex is unclear and does not promote a greater understanding of the concept. Specifically, his Figures 103 and 104 could be more easily understood if one aspect of Kh, subgroup inclusion, was mentioned.

In Chapter 2, it was stated that a non-IBI, or non-IBT, n-group usually has an (n-1)-group distribution of singletons, i.e., it contains n different (n-1)-groups. IBI, or IBT, n-groups have a smaller number of different (n-1)-groups because some of their subgroups are multiply presented. Therefore, it would seem that the size of Kh, which depends on the number of different n-groups contained in it, should reflect the difference between those n-groups (based as referential groups for Kh) that are IBI or IBT, and those that are not. Figure 103, which indicates Kh sizes for most n-groups, can be modified (Table IX) to indicate the size of Kh of IBI, IBT, and non-IBI or non-IBT, n-groups. Forte does not list the Kh size for 4-23, which is twelve. It can be seen that IBI and IBT n-groups have the smallest Kh sizes, and that non-IBI and non-IBT n-groups have the largest.

TABLE IX

Subcomplex Sizes

Cardinal Number	IBI and/or IBT N-groups			Non-IBI and Non-IBT N-groups
	IBI and IBT	IBT	IBI	
3	20		40 43 44	62 63 64
4	4 8 10		12 13 14	20 24 25 26
5			7 9 10 11	13 14 15 16 17 18
6	7 9 13	19	16 20	23 26 28 30 31
6 (Z-pairs)			10 11	15 16 17 21

The same segmentation of n-groups can be employed to modify Forte's Figure 104 (Table X), which indicates the number of invariant subgroups for each n-group. In Figure 104, pc set 4-19 is incorrectly listed as having three invariant subsets. 4-19 (4:15) has a 3-group content of 3:3 4 11 [$\underline{12}$] and a 2-group content of 2:$\underline{1}$ $\underline{3}$ $\underline{4}^3$ $\underline{5}$. It, therefore, has four invariant subsets: 3:[$\underline{12}$], 2:$\underline{1}$, 2:$\underline{3}$, and 2:$\underline{5}$.

TABLE X

Number of Invariant Subgroups

Cardinal Number	IBI and/or IBT N-groups			Non-IBI and Non-IBT N-groups
	IBI and IBT	IBT	IBI	
8	1 3 4		5 6	13 16 17 18
7			4 5 6	10 11 12 13 14 15 16
6	0 1 2	4	4 5 6	7 8 10 11 12 13 14 15
5			3 4 5	5 6 7 8 9
4	0 1		2 3	4 5 6

The previous two examples have attempted to illustrate weaknesses in Forte's presentation. It should be mentioned that Figure 104 and Forte's discussion of it is peripheral to Kh, as are other topics dealing with the subcomplex.

The following chapter discusses the set complex in more detail because it forms the basis of Forte's analytical method. From a theoretical perspective, K and Kh have one major fault; they are concerned with complementation in reference to 12:[1] only. In applying K and Kh to analysis, context should determine the choice of a referential n-group. In theory, however, the restriction of K and Kh to 12:[1] is arbitrary. The previous chapter demonstrated that complementation, with respect to other n-groups, is responsible for some interesting relations occurring, i.e., Z-pairs. The extension of the referential n-group from 12:[1] to other n-groups can be justified, if not by context in analysis, then by theory for its own sake.

CHAPTER 7

ANALYSIS AND CONCLUSION

Summary and Introduction

The preceding chapters have dealt exclusively with theory. Although problems and weaknesses in Forte's concepts have been pointed out, and suggestions for their improvement discussed, no mention as to the validity of this theory has been stated. Such a statement can only be verified after the theory has been applied to analysis. Not only is this chapter concerned with analysis, but it also contains an appraisal of the set theoretic approach, as practised by Forte, in the analysis of atonal music.

Forte's analytical procedure can best be evaluated after pieces, which he analyses, are re-examined and the resulting conclusions compared. The following two sections each contain an original analysis (by the writer) of a piece that Forte examines, a brief summary of Forte's analysis, and an evaluation.[1] The final section reviews these remarks, and considers other aspects of Forte's approach in order to ev evaluate its significance as an analytical procedure.

1 The music discussed in these two sections is contained in Appendices 2 and 3, respectively.

The foundation of Forte's analytical method is the set complex (K) and subcomplex (Kh). The method begins with a segmentation of the piece and the subsequent designation of these pc sets by name, e.g., 4-28, 6-Z6, etc.. These are then listed as an array in which a "K" or "Kh" is located to indicate that one of these relations exists between the pc sets aligned with it (row and column). Those pc sets that are K or Kh related to the largest number of represented pc sets in the table (array) are considered more important strucurally than those pc sets that have a fewer number of pc sets related to them. The presence of one of these significant pc sets in separate sections of a piece is employed as a measure of similarity and connectedness between them. Not only does this measure depend on the division of the piece into smaller sections, but also on the segmentation process that determines the pc sets which are compared to establish K or Kh membership. The latter procedure is very subjective because of the large number of possible segments that can be extracted from a musical context. Weaknesses in the application of this method will be encountered in subsequent sections.

Forte's designation of specific pcs by integers will be employed in the analyses. Thus, "0" is assigned to all enharmonic equivalents of "C" regardless of register. "1" represents all enharmonic equivalents of "C#", and so on.

Alban Berg's Opus 4/3

Alban Berg's Opus 4/3 is a short piece for orchestra and solo voice which can be divided into four sections for the purpose of analysis. These sections are identical to those of Forte: A (mm. 1-8), B (mm. 8-11), C (mm. 11-17), and D (mm. 18-25). The subsequent analysis will demonstrate that interrelations between individual sections and their pc content are dependent on linear motion toward, or away from, specific pcs. Linear motion is defined as descending or ascending movement by numerically adjacent pcs. Therefore, linear motion may not be associated with chromatic motion in the traditional sense, since with the use of integer notation there is no indication of register. Pcs 4-5-6-7 and 11-10-9 are examples of linear ascent and descent, respectively.

The orchestral part in A is composed of five <u>tutti</u> attacks of a vertical sonority which contains all twelve pcs. These attacks occur at intervals of four quarter-notes. While the chord is repeated, the woodwind and brass parts change pcs, but the registral structure of the chord remains unaltered. After the fifth attack, the pcs of the chord are removed one at a time at temporal intervals of one eighth-note, beginning with the lowest pc and proceeding upward in register. This order of releases is identical to the registral voicing of the chord (Figure 19).

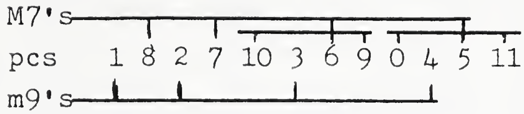

Figure 19. Registral structure of the orchestral chord

The segmentation in Figure 19 produces an interesting division. Pcs which are not part of the conjunct M7's (8-7-6-5) or m9's (1-2-3-4) constitute a vertical representation of the BACH motive (pcs 10-9-0-11). Berg's use of <u>soggetto cavato</u> in his <u>Chamber Symphony</u> and operas has been noted on numerous occasions. The presence of the BACH motive in this context may be coincidental because further use of these pcs or the technique itself could not be found elsewhere in the piece. These four pcs are a chromatic segment (pcs 9-10-11-0), as are the conjunct m9's and M7's: all 4:<u>1</u>'s. The twelve pcs have, therefore, been segmented into three 4:<u>1</u>'s by motivic (BACH?) and intervallic (conjunct M7's and m9's) relations.

In A, particular pcs in the vocal part attain structural importance because they are stressed more than the other pcs. The nature of this stress can be melodic, metric, or based on duration. Metric emphasis is caused by the orchestral chord which recurs every four quarter-notes. Pcs in the vocal part that coincide with these attacks are metrically stressed because they are situated on the strong beat of a metric pattern. Pcs 6-7-10-1 coincide with these attacks (Figure 20).

measure	1	2	3	4	5	6	7	8
duration in quarter-notes		1 1 1	1 1 1	2 2	1 1	1 $2\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$1\frac{1}{2}$
pcs		7 6 5	9 8 7	1 10	10 8 2	11 1	1 5 2 7 3 9	0
orchestral meter	X	X	X		X	X		

Figure 20. Reduction of the vocal part in A

Those pcs that have a duration greater than one quarter-note are 0-1-10 (Figure 20). Melodic stress is based on relations between the pcs themselves. As framing pcs, the initial and final notes of the vocal part are structurally important: pcs 7-0. Pc7 is further emphasized by linear motion among the first six pcs (Figure 21). The registral extremes of the orchestral chord are pcs 1-11 which are the leading tones of pc0, the final pc in the vocal part of A. This symmetry is literal because pc0 is exactly in the center of the chord, being a M14 (octave plus a M7) from each of its leading tones.

pcs 7 6 5 9 8 7

linear motion away from pc7 — linear motion toward pc7

Figure 21. Melodic emphsis of pc7

Since pc4 is the only pc not present in the vocal part of A, its omission is therefore noteworthy. Indeed, section B includes linear motion about pc4 and the vocal part in C stresses pc4, therefore, its exclusion in A (vocal part) tends to balance the importance it acquires in other sections.

The pcs in the vocal part that have been designated as important can be arranged in two groups (Figure 22). The upper row of pcs form a 4:[29] (pcs 1-4-7-10) and the lower row of pcs form a 2:[6] (part of another 4:[29]). The orchestral chord includes a 4:[29] as four registrally adjacent pcs (3-6-9-0). Examination of sections B and C will indicate that a 4:[29], or the pcs in it, are presented in a more foreground role than they are in A, in which their role is of a background nature.

melodic	duration	metric	exclusion
7	1-10	7-10-1	4
0	0	6	

Figure 22. Summary of the emphasized pcs in the vocal part of A

The pc content of B can be segmented into three levels of structure: (1) the pcs of the vocal part, occasionally doubled in the orchestral parts; (2) those pcs contained in two chords which provide a harmonic background; and (3) the pcs in the remaining parts, which consist of linear motion resulting in a 4:[29].

The vocal part in B consists of pcs 5–9, which are articulated within a triplet-eighth rhythm. These pcs are also present in the oboe and english horn parts. The latter part even employs the same rhythm as the voice, which occurs after the final pc in the vocal part (Figure 23). The english horn part concludes with pc10. This is included in Figure 23 because it is preceded by pc9. Furthermore, pc9 occurs on each beat of m. 10, which increases the function of pc9 as a leading tone of pc10 that is attacked on the first beat of m. 11. Pcs 5–9 are symmetrical about pc7 (Figure 21), therefore, just as pc9 functions as a leading tone to pc10, pc5 also functions as a leading tone of pc4 (pcs 4–10 are also symmetrical about pc7). The leading tone function of pc5, however, is not present on this level of structure. The pcs in the oboe part, as indicated, are merely a doubling of the pcs in the vocal part.

measure	9	10			11
beat	3	1	2	3	1
english horn				5 9	10
oboe		5	9		
voice	5 5 5	9 5 5	9 – 5	9	

Figure 23. Reduction of the vocal part in B

Two vertical sonorities, which are transpositions of each other, provide a harmonic background in B. The transposition relation is literal because the second chord (pcs 5-11-3) is a M6 below the first chord (pcs 2-8-0). These chords are attacked in an order involving two appearances of the first chord and one of the second chord (Figure 24). This three chord pattern is repeated, but with a modification; the second chord of the pattern is not present, but the oboe part contains two of its pcs. All these chords are attacked at intervals of one quarter-note, and the two pcs in the oboe part coincide in time with the expected chord. These pcs are followed by pc3, which is a doubling of one of the pcs in the final chord. The last two pcs in the oboe part are the upper pcs in each chord (pcs 0-3) and, hence, serve to emphasize the M6 relation between the two chords. This M6 is followed by two other M6's whose pcs form a 4:[29], and so the oboe part functions in a preparatory role. By means of segmentation, these two chords can be seen to be composed of the previously mentioned M6 (pcs 0-3) plus a 4:[29] (pcs 2-5-8-11).

	flutes	french horns		strings	oboe	piano
	0	0	3	0		0—3
	8	8	11	8	8	11
	2	2	5	2		5
quarter-note intervals	X	X	X	X	X	X
measure	9		10			11

Figure 24. Reduction of the harmonic background in B

Doubling of pcs in these chords accounts for pcs in other orchestral parts. For example, pc5 on the second eighth of m. 10 (french horn) is doubled in the voice, oboe, and clarinet parts (Figure 23).

The remaining pcs in B can be divided into two groups. The first group includes pcs involved in linear motion about pcs 1-4. At the end of B, two M6's form a 4:[$\underline{29}$] (pcs 1-4-7-10), which later becomes the foundation of the orchestral part in C. This second group of pcs (4:[$\underline{29}$]) contains the important pcs of the first group (pcs 1-4).

The clarinet part in mm. 9-10 involves linear motion toward pc4 (Figure 25): pcs 2-3 and 6-5. Pc4 is not realized, however, but the oboe part deflects the motion of the first two pcs (when it doubles pc3 of the clarinet part) and descends chromatically to pc1 (Figure 25). The following pcs in the oboe part are a doubling of pcs in the vocal part (Figure 23), and while pc5 (clarinet) is functioning as one (of two) upper leading tones of pc4, it is also a doubling of a pc in the harmonic background (m. 10, french horn).

oboe		3		2	1	5	9
clarinet	2	3	6			5	
measure	9				10		

Figure 25. Reduction of the oboe and clarinet parts in mm. 9-10

The two M6's in m. 11 form a 4:[29]. The first M6 (trombone) overlaps the M6 in the oboe part of m. 11, as indicated in Figure 26. In reference to Figure 23, pc10 was stated as being the resolution of pc9, this pc10 precedes the attack of pc10 in the trombone part by one sixteenth-note. Similarly, the initial note of the second M6 (pc4) is prepared by linear motion (Figure 25). Therefore, the first pc of each of the two M6's is prepared by a leading tone. All these pcs are symmetrical about pcs 1-7 (Figure 27).

oboe 0——3--------M6
trombone 10——1------M6
lower strings 4-7-M6————4:[29]

Figure 26. Reduction of descending M6's in m. 11

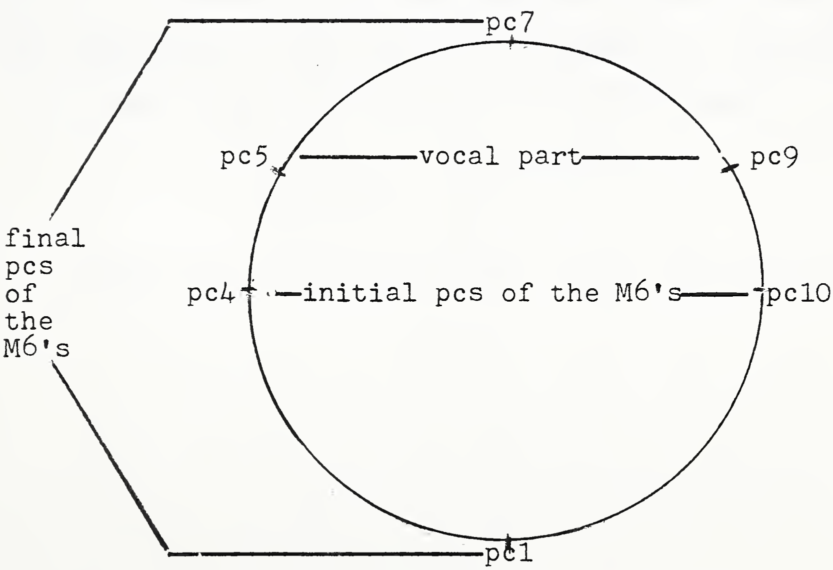

Figure 27. Symmetry of emphasized pcs in B

The final M6 contains pcs 4-7, which were important pcs in A. Pc4 is introduced here for the first time, apart from its inclusion in the orchestral chord of A. Pc7, being the final pc of B, has added significance of a melodic nature which is analogous, in a symmetrical manner, to A because pc7 was the initial pc of the vocal part in A.

- The vocal part in C is a transposition of the vocal part in A (Figure 28). In addition, the stress of pc4 complements pcs 1-7-10 (in A) within the context of a 4:[29]. Besides transpositional identity, the vocal part in A and C are also identical with respect to their final note: pc0. The penultimate pcs are symmetrical about the final pc. In A, the approach to pc0 is made by pc9, while in C, the penultimate note is pc3. These three pcs (9-0-3) when combined with the highest pc in the vocal part (pc6) form a 4:[29]. The appearance of pc0 an octave higher in C than in A will have significance when section D is discussed.

section A	7	6	5	9	8	7	.	. .
section C	4	3	2	6	5		3	0

Figure 28. Comparison of the vocal parts in A and C

The orchestral parts in C emphasize the pcs of a 4:[29] which contains pcs 1-4-7-10. This sonority is introduced at the end of the previous section, and is sustained and prolonged throughout C (Figure 29). In the traditional sense, chromatic movement is present in mm. 13-16, as pcs 4-7 replace one another as registral extremes in these two lines, which begin a M6 apart (the same register as the final M6 in B) and expand by the end of m. 15 to a m10. In m. 16, this chromatic movement is continued and now involves pc10 descending to pc9. Pc1 remains stationary and coincides exactly with the initial pc of D

measure	11	12	13	14	15	16
	4	4 - 4 - 4 -	5 - 5	- 6 -	7 - 7	8
		7 7 - 7 - 7	- 6 -	5 - 5	- 4 -	3
		10 -	- - -	- - -	- - -	9
		1 -	- - -	- - -	- - -	1

Figure 29. Reduction of the orchestral part in C

Section D complements and recapitulates A; in addition, it continues a progression from A and C. The pcs in the vocal part of D are, with one exception, identical in register and rhythm to the pcs in the vocal line in A. Pc0, the final pc in each section, is the exception because it is two octaves higher in D than in A. This is significant because the orchestral chord in D, which is identical in pc structure to the chord in A, is also two octaves higher than its appearance in the opening section. Therefore, the symmetry between the registral extremes of the orchestral chord and the final pc of the vocal part in A is also maintained in D. This two octave increase in register is bridged in C, where pc0 is one octave from the pc0 in sections A and D. Pc0 is present only once in the vocal part in each of sections A, C, and D. Its location at the end of each section and its registral placement, throughout the piece, creates structural unity and development.

The orchestral chord in D is articulated in such a manner, that at equal temporal intervals (dotted eighth-notes) a different pc is attacked and sustained. The order of pcs in this attack sequence is identical to the order of releases of pcs in the orchestral chord of A. This order begins with the lowest pc and increases in register (Figure 30). The orchestration is also complementary to A: woodwinds and brass in A; and strings, celeste, and cymbal in D. Another similarity between A and D involves the vocal line in relation to the orchestral chord at the moment when the first pc is released (pc1) in m. 6 (A), and when the last pc (pc11) is attacked in m. 24 (D). At both these locations, the vocal part has the same note (pc1). This can not be regarded as coincidental, but must be considered as a reinforcement or confirmation of the fact that pc1, in addition to pcs 4-7-10 (4:[29]), increases the degree of unity between sections A and D.

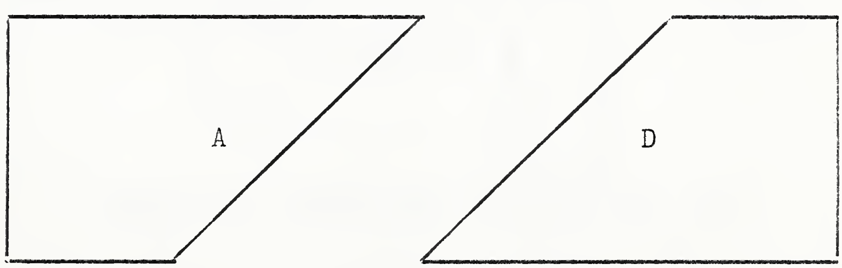

Figure 30. Registral representation of the orchestral chords in sections A and D

In the foregoing analysis, 4:[29] and its pcs were crucial to an understanding of Berg's Op. 4/3. The pcs of a 4:[29], if not stated explicitly, were inferred by linear motion. Figure 31 is a summary of the pcs which were influential in the previous analysis. O and V represent orchestral and vocal parts, respectively. The enclosed pcs are 4:[29]'s. One other 4:[29] is embedded within the orchestral chords of A and D (pcs 0-3-6-9). Therefore, all three 4:[29]'s are present in the piece, although they are not of equal structural significance. Figure 31 is not meant to be all encompassing. It does not include linear motion, nor does it include mention of the transposition, by octaves, of pc0. Instead, it is intended as a general statement in anticipation of the discussion of Forte's analysis.

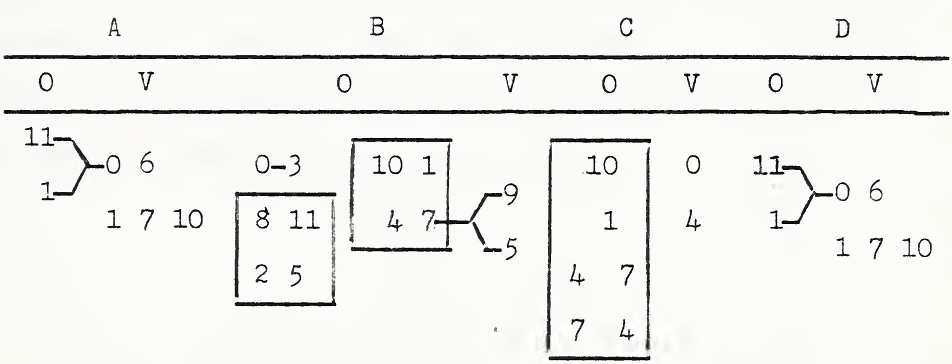

Figure 31. Reduction of Berg's Op. 4/3

Despite the fundamentality of 4:[29] to an understanding of the piece in the previous analysis, Forte's analysis does not even indicate this 4-group (4-28) in his segmentation. Although some of the examples of 4:[29] cited are implicit, in mm. 11, 12, and 15, this 4-group is more explicitly articulated and it is surprising that Forte overlooked their presence.

If, indeed, 4-28 was not simply overlooked, its deliberate omission in the segmentation is convenient for Forte's method. 4-28 has only four pc sets in its subcomplex (Kh). Even if Forte indicated the presence of a 4-28, not to mention its musical significance, it would not be deemed important by his analytical method because many other pc sets would have a larger number of represented pc sets in their set complex or subcomplex. It can not be ascertained whether the omission of any 4-28 was deliberate or an oversight. In any event, it avoided a conflict between theory and musical sensibility.

In his analysis, Forte includes two segmentations of the vocal part based on independence or dependence of the text. The musical significance of pcs 7-6-5 and 9-8-7 (both 3-1's) was mentioned in the previous analysis. In the text-dependent segmentation, Forte indicates only one 3-group, pcs 10-8-2. Surprisingly, this segment, which is text-dependent, is underlayed by two words. The two 3-groups just mentioned are also underlayed by two words, but are not present in the segmentation. Syntactically speaking, Forte's segmentation of the vocal part in section A is unjustified (Figure 32). An alternate segmentation is shown in Figure 32 with the pc sets depicted, as in Forte's segmentation. Besides being of questionable validity, a product of his segmentation (3-8) is not included in the table where K and Kh relations are listed. To conclude, no apparent reason can be determined to explain Forte's text-dependent segmentation.

Forte's segmentation

8-21

6-Z37 3-8 6-21 2-3

Über die Grenzen des All blicktest du sinnend hinaus;

5-13 4-13 8-21

alternate segmentation

Figure 32. Text-dependent segmentation of the vocal part in A

Forte's appendices include a list of pc sets that are Kh related to pc sets of cardinal number four, five, and six. There should be a list indicating the pc sets that are Kh related to pc sets of cardinal number three, especially when 3-groups are included within the Kh list of the other pc sets. In addition, there should be a list of K relations because they are used and mentioned by Forte in his analyses concurrent with Kh relations. Although Kh implies K, the converse is not true.

Berg's Altenberg Lieder (Opus 4/3) may have been an unwise choice, by Forte, to illustrate the application of his theory. No sense of continuity or connectedness is apparent in the analysis, nor are one or more pc sets stated as being structural throughout the piece. Even with the employment of a subjective segmentation procedure which results in the omission of musically appropriate segments, and the inclusion of unconvincing examples, the end result of the analysis is vague and inconclusive. Although a few weaknesses in Forte's analytical method have been described, the following section will uncover problems that will diminish the credibility of this method as a legitimate theory for analysis.

Anton Webern's Opus 7/3

For the purpose of analysis, Webern's short piece for violin and piano is divided into four sections. With one exception, these divisions are identical to those employed by Forte: A (mm. 1-5), B (mm. 5-9), C (mm. 9-11), and D (mm. 12-14). The exception is the division between C and D. This will be discussed in connection with Forte's analysis.

This analysis will illustrate that linear motion and the overall pc content of sections are related to the pc content of the three disjunct 4:[29]'s or the 4-groups containing the following pcs: 0-3-6-9, 1-4-7-10, and 2-5-8-11.

The total pc content of A consists of two disjunct 3:1's (pcs 1-2-3, and 8-9-10). In terms of linear motion, pcs 2-9 are essential because each pc has its upper and lower leading tone present. Examples of linear motion about pc9 are found in mm. 2-3 (pcs 10-8 sounding with pc9), and m. 4 (pcs 8-10-9 in the piano part). Pc2 is similarly introduced by pcs 1-3 in m. 4. Pcs 2-9 are also the first and final pcs attacked in this section, however, pc3 is the final sounding pc in A (Figure 33).

pcs \ measure	1	2	3	4	5
10	P P	P		V P –	
9	V – – –	– –	–	V P	
8			P –	– – – –	
3				V P –	– –
2				V	P
1				P	

Figure 33. Reduction of the piano (P) and violin (V) parts in A

The last six notes in the piano part can be divided into two 3-groups that are inversions of each other. In addition, the final notes of these segments are pcs 2-9 (Figure 34). Another example of inversion involves a segment that overlaps the preceding segmentation and includes a segment of the violin part (Figure 34). The piano segment is justified because it is articulated by the left hand. Not only are these segments related by inversion, but their initial and final notes are pcs 2-9.

P   V   P

8 10 9   3 1 2   3 2 10 9   9 1 2

Figure 34. Examples of inversion-related segments in A

Pc3 is the final sounding pc in A in addition to being the highest pc in the section (P, mm. 4-5). It is also the highest pc in the violin part of this section (m. 4).

All the preceding observations have stressed the importance of pcs 2-3-9. The statement in the introduction, about the pc content of sections being related to the three 4:[29]'s, is exemplified in A. Pcs 3-9, 2-8, and 1-10, each belong to a different 4:[29]. The melodic contour of the violin begins with pc9, ascends to pc3 and then descends to pc9. This strengthens the previous segmentation according to disjunct 4:[29]'s; however, the importance of pcs 2-9 cannot be overlooked in this section. It is in section B, that 4:[29]'s become prominent.

The violin part in B consists of two 4-note figures that are related by transposition (Figure 35a). They are alternated and repeated throughout B. The pc content of these two figures is an 8:[29]. A segmentation of these eight pcs illustrates that they contain two disjunct 4:[29]'s (the complement of 8:[29]) as shown in Figure 35b.

9 2 3 8   0 5 6 11

m3

a. The transpositional relation

4:[29]

9 2 3 8   0 5 6 11

4:[29]

b. The 4:[29] content

Figure 35. Reduction of the violin part in B

Pcs that are part of a 4:[29] are emphasized in the piano part of D (Figure 36). Pcs 11-5 are the initial and final sounding pcs of B, respectively. The commencement and termination of pc5 (sustained through most of B) is accompanied by pcs 4-6, respectively: leading tones of pc5. The remaining pcs in the piano part include pc2 and its leading tones (pcs 1-3). Excluding these leading tones, the piano part in B reduces to pcs 11-2-5, which are part of one 4:[29]. In addition, pc2 occurs on the downbeat of m. 7 which coincides with pc11 in the violin part and, of course, the pedal throughout B (pc5). The down beat of m. 7 is the exact beginning of the second half of the piece, as measured in eighth-notes. It is also at the height of a crescendo.

measure	5	6	7	8
pcs 11	-			
6			-	-
5	-	- - -	- -	- -
4		-		
3		-		
2			-	
1		-		

Figure 36. Reduction of the piano part in B

The registral extremes of the two violin figures in B are pcs 9-8 and 0-11, respectively. These four pcs are the two upper and lower leading tones of pc10, which is not present in B. The total pc content of C consists of these four leading tones plus their note of resolution (pc10), which occurs as the lowest note in a 3-note chord (pcs 10-11-0) composed of two conjunct m2's. This connection between B and C is strongly represented because the final violin figure in B has pcs 9-8 as its registral extremes, and these are the only pcs in the violin part of C. Out of context, another interpretation of the 3-note chord is possible; pc11 is stressed because it is surrounded by its upper and lower leading tones. Both are a m2 from pc11. This provides another connection with B because pc11 is the initial note of that section. C, as a whole, has another interpretation; the violin tremolo (pcs 9-8) emphasizes pc9 because it is the highest note in C. Pc 8 is the lowest note in C and alternates with pc10 (in the piano part) as the lowest pc at any time. These two pcs are leading tones of pc9. Section C is an excellent example illustrating the difficulties in designating one particular pc as more important structurally than another pc.

The pc content of D can be segmented into two 3-groups that are identical (3:$\underline{10}$'s) and literal transpositions of one another (Figure 37). The registral extremes of this chord (pcs 3-1) are leading tones of pc2. Inclusion of pc7 within the final chord is significant because it is the only instance of a pc7 in the entire piece.

$$\underbrace{3\ 6\ 0}\quad \underbrace{4\ 7\ 1}$$

m9

Figure 37. Reduction of the pcs in D

Before discussing relations between individual sections (with respect to pc content), texture will be discussed briefly. Texture is a measure or indication of the number of pcs associated with a simultaneous attack or overlapping parts. The texture in each succeeding section increases in complexity (Figure 38). In C, for example, the maximum number of pcs in a simultaneous attack is three (piano in m. 9), and the maximum number of pcs sustained at any one time is four (the 3-note chord plus one note from the violin part).

section	A	B	C	D
maximum number of pcs in a simultaneous attack	1	2	3	5
maximum number of pcs in sustained parts	2	3	4	6

Figure 38. Levels of texture

The emphasized pcs in each section often complement each other when viewed from the perspective of 4:[$\underline{29}$]'s. Sections C and D offer the best example of this connection. The pc content of D consists of two 3:$\underline{10}$'s, which is the only 3-group that is an (n-1) subgroup of 4:[$\underline{29}$]. The lower 3:$\underline{10}$ plus pc9 produces a 4:[$\underline{29}$]; pc9 is the highest and final pc in the violin part of C. The upper 3:$\underline{10}$ plus pc10 also form a 4:[$\underline{29}$]; the lowest note in the 3-note chord of C is pc10. This complementation within 4:[$\underline{29}$]'s implies that C and D, as one section, strongly articulate two 4:[$\underline{29}$]'s.

On the basis of articulation and register, sections C and D are also very similar. In terms of register, C and D have a narrow registral range, at least when compared with B. Each section consists of a sustained chord plus a 2-note figure articulated twice. In C, the violin tremolo is repeated twice around the piano's 3-note chord. In D, the pcs of the tremolo figure in the piano part are repeated, but as a 2-note chord (mm. 13-14). Combining C and D within one section gives the piece an overall form of A-B-A' with respect to register. In terms of duration, these sections are nearly equal in length: sixteen, nine, and fifteen eighth-notes, respectively. The brevity of B is balanced by its more intense rhythmic activity of triplet sixteenth-notes throughout most of the section (violin part).

There are also direct parallels between sections. The first two pcs of the violin part in B are also the initial and final pcs attacked in A. In addition, the first three pcs of the violin part in A and B are identical and in the same register, although the order is different. Finally, the registral extremes of the violin figures in B are contained in C along with pc10, the resolution of pcs 8-9-11-0.

Although this analysis has stated the importance of 4:[<u>29</u>] (4-28), Forte's analysis does not indicate it in his segmentation. This is not surprising because, as the previous analysis showed, 4:[<u>29</u>] is not explicitly articulated.

In Forte's segmentation and the determination of K and Kh relations, three pc sets are designated as important: 6-Z6, 6-Z13, and 4-9. 6-Z6 is the pc content of A; 6-Z13 is the final sonority in D; and 4-9 is the 4-note violin figure in B. There is no disputing the importance of these pc sets within the context in which they have previously been described. The problem with Forte's analysis is that his segmentation is directed toward finding these pc sets in order that a measure of continuity can be established. In one case, this leads to a musically questionable segmentation, and the omission of a musically logical segmentation. These two segments are in mm. 8-11.

Forte's segmentation places the fifth articulation of the 4-note violin figure in mm. 8-9 at the beginning of C. There is no musical justification for segregating this figure from the preceding four. In the segmentation of C, Forte indicates a 6-Z13. This is produced by including the pcs of the violin figure (mm. 8-9) and pcs 11-0 from the 3-note chord (mm. 9-11). What is the justification for this segmentation? Why is pc10 (part of the 3-note chord) excluded? The most obvious reason is that the end justifies the means. In other words, the segmentation that produces a 6-Z13 supplies a connecting link to D, whose entire pc content is a 6-Z13. Another problem with the segmentation is that no mention of the total pc content in mm. 10-11 is included. Segmentation that is based on musical priorities would certainly include this as a segment, but Forte's priorities are otherwise. The pc content of mm. 10-11 is a 5-1. This pc set is not K or Kh related to any of the three prominent pc sets already mentioned, including 6-Z13. Therefore, if 5-1 were present in Forte's segmentation of C, the criteria he uses for analysis would not yield a connection between this section and D.

To summarize, the segmentation of C by Forte reveals a flaw in his method. If certain pc sets are not present in individual sections, there is no basis for a connection between them. This will, obviously, not produce a satisfactory analysis. This flaw, however, is hidden by Forte's segmentation which neglects musically appropriate segments and includes, in its place, dubious examples. The practice of finding data to suit the theory, when it involves the deliberate omission of relevant material, is untenable. When this practice is necessary in the application of a theory or method to produce meaningful results, that theory surely lacks credibility.

Conclusion

At this point, preceding criticisms dealing with Forte's analyses require summation, and comments of a general nature deserve mention. Firstly, the basis of his analytical method is the set complex or subcomplex. Both of these concepts employ the principles of inclusion and complementation within the 12-group. In the Webern example, all twelve pcs were not articulated until the final chord. The possibility of employing other groups as the referential pc set in K or Kh relations should be determined by context. In any event, Forte makes use of only the 12-group. Z-related pairs are significant, in a complementary sense, when other n-groups are the foundation of complementary relations. This extension was discussed in Chapter 5.

Comment must be given to Forte's appendices. Firstly, there is no appendix indicating K relations, although one for Kh relations is included. Such an appendix should be provided since both relations are shown throughout the text as the final stage of the analytic process. Secondly, a listing of Kh (and K) relations for 3-groups in succession should also be included because they are listed with pc sets that are Kh related to pc sets of cardinal number four, five, and six.

Any segmentation process, because of the overwhelming complexity if done completely, must depend on analytical decisions in order to obtain a manageable quantity of pc sets. Forte's segmentation process is not based entirely on analytical decisions. The determination of segments by Forte is often biased in favour of obtaining segments that will allow him to claim that a connecting relation exists between sections. This biased approach results in segments that are often musically indefensible, and omits other segments that are musically appropriate because they do not conform to Forte's theory.

Although the two detailed analyses have utilized linear motion and 4:[29] in their discussion, it is not suggested that the analysis of atonal music should proceed along these lines. Certainly, it is unlikely that a theory of atonal music would incorporate the diminished-seventh chord to any great extent. Linear motion may have historical precedence, but to incorporate it into a theory would require its role in the analyses of a large number of works in the atonal idiom. Although a great number of pieces have been examined, there has not been a consistent approach or methodology that would enable a theory to be extracted.

Forte's theory is based only on unordered pitch structures and his analyses achieve varying degrees of success. It is suggested that any analysis be founded on parameters other than pitch. A theory that may eventually elucidate atonal music should include a discussion of structure with respect to other concepts besides pitch: e.g., register, rhythm, and timbre.

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APPENDIX 1

LIST OF N-GROUPS

In Chapter 2, the advantages of ordering and describing n-groups by means of (n-1)-group contents was discussed. Table XI lists the (n-1)-group contents for n-groups from cardinal number three through twelve, inclusive. IBI groups are indicated by a "_" beneath their ordinal number. IBT groups are indicated by a "[ ]" around their ordinal number. Table XII, as a cross-reference, is intended to be used in conjunction with Forte's text.

TABLE XI

Pcs and (n-1)-group Contents of N-groups

N-group	Pcs	(n-1)-group Content
2:$\underline{1}$	(0,1)	
2:$\underline{2}$	(0,2)	
2:$\underline{3}$	(0,3)	
2:$\underline{4}$	(0,4)	
2:$\underline{5}$	(0,5)	
2:[$\underline{6}$]	(0,6)	
3:$\underline{1}$	(0,1,2)	2:$\underline{1}^2$ $\underline{2}$
3:2	(0,1,3)	2:$\underline{1}$ $\underline{2}$ $\underline{3}$
3:3	(0,1,4)	2:$\underline{1}$ $\underline{3}$ $\underline{4}$
3:4	(0,1,5)	2:$\underline{1}$ $\underline{4}$ $\underline{5}$
3:5	(0,1,6)	2:$\underline{1}$ $\underline{5}$ [$\underline{6}$]
3:$\underline{6}$	(0,2,4)	2:$\underline{2}^2$ $\underline{4}$
3:7	(0,2,5)	2:$\underline{2}$ $\underline{3}$ $\underline{5}$
3:8	(0,2,6)	2:$\underline{2}$ $\underline{4}$ [$\underline{6}$]
3:$\underline{9}$	(0,2,7)	2:$\underline{2}$ $\underline{5}^2$
3:$\underline{10}$	(0,3,6)	2:$\underline{3}^2$ [$\underline{6}$]
3:11	(0,3,7)	2:$\underline{3}$ $\underline{4}$ $\underline{5}$
3:[$\underline{12}$]	(0,4,8)	2:$\underline{4}^3$
4:$\underline{1}$	(0,1,2,3)	3:$\underline{1}^2$ 2^2
4:2	(0,1,2,4)	3:$\underline{1}$ 2 3 $\underline{6}$
4:3	(0,1,2,5)	3:$\underline{1}$ 3 4 7
4:4	(0,1,2,6)	3:$\underline{1}$ 4 5 8

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
$4{:}\underline{5}$	(0,1,2,7)	$3{:}\underline{1}\ 5^2\ \underline{9}$
$4{:}\underline{6}$	(0,1,3,4)	$3{:}2^2\ 3^2$
$4{:}\underline{7}$	(0,2,3,5)	$3{:}2^2\ 7^2$
4:8	(0,2,3,6)	$3{:}2\ 3\ 8\ \underline{10}$
4:9	(0,1,3,5)	$3{:}2\ 4\ \underline{6}\ 7$
4:10	(0,2,3,7)	$3{:}2\ 4\ \underline{9}\ 11$
4:11	(0,1,3,6)	$3{:}2\ 5\ 7\ \underline{10}$
4:12	(0,1,3,7)	3:2 5 8 11
$4{:}\underline{13}$	(0,1,4,5)	$3{:}3^2\ 4^2$
$4{:}\underline{14}$	(0,3,4,7)	$3{:}3^2\ 11^2$
4:15	(0,1,4,8)	$3{:}3\ 4\ 11\ [\underline{12}]$
4:16	(0,1,4,6)	3:3 5 7 8
4:17	(0,1,4,7)	$3{:}3\ 5\ \underline{10}\ 11$
$4{:}\underline{18}$	(0,1,5,6)	$3{:}4^2\ 5^2$
$4{:}\underline{19}$	(0,1,5,8)	$3{:}4^2\ 11^2$
4:20	(0,1,5,7)	$3{:}4\ 5\ 8\ \underline{9}$
$4{:}[\underline{21}]$	(0,1,6,7)	$3{:}5^4$
$4{:}\underline{22}$	(0,2,4,6)	$3{:}\underline{6}^2\ 8^2$
4:23	(0,2,4,7)	$3{:}\underline{6}\ 7\ \underline{9}\ 11$
$4{:}\underline{24}$	(0,2,4,8)	$3{:}\underline{6}\ 8^2\ [\underline{12}]$
$4{:}\underline{25}$	(0,2,5,7)	$3{:}7^2\ \underline{9}^2$
$4{:}\underline{26}$	(0,3,5,8)	$3{:}7^2\ 11^2$
4:27	(0,2,5,8)	$3{:}7\ 8\ \underline{10}\ 11$
$4{:}[\underline{28}]$	(0,2,6,8)	$3{:}8^4$

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
4:[$\underline{29}$]	(0,3,6,9)	3:$\underline{10}^4$
5:$\underline{1}$	(0,1,2,3,4)	4:$\underline{1}^2$ 2^2 $\underline{6}$
5:2	(0,1,2,3,5)	4:$\underline{1}$ 2 3 $\underline{7}$ 9
5:3	(0,1,2,3,6)	4:$\underline{1}$ 3 4 8 11
5:4	(0,1,2,3,7)	4:$\underline{1}$ 4 $\underline{5}$ 10 12
5:$\underline{5}$	(0,2,3,4,6)	4:2^2 8^2 $\underline{22}$
5:6	(0,1,2,4,5)	4:2 3 $\underline{6}$ 9 $\underline{13}$
5:7	(0,2,3,4,7)	4:2 3 10 $\underline{14}$ 23
5:8	(0,1,2,4,6)	4:2 4 9 16 $\underline{22}$
5:9	(0,1,2,4,8)	4:2 4 12 15 $\underline{24}$
5:10	(0,1,2,4,7)	4:2 $\underline{5}$ 11 17 23
5:$\underline{11}$	(0,3,4,5,8)	4:3^2 15^2 $\underline{26}$
5:12	(0,1,2,5,6)	4:3 4 $\underline{13}$ 16 $\underline{18}$
5:13	(0,1,2,5,8)	4:3 4 17 $\underline{19}$ 27
5:14	(0,1,2,5,7)	4:3 $\underline{5}$ 16 20 25
5:$\underline{15}$	(0,1,2,6,8)	4:4^2 20^2 [$\underline{28}$]
5:16	(0,1,2,6,7)	4:4 $\underline{5}$ $\underline{18}$ 20 [$\underline{21}$]
5:17	(0,1,3,4,6)	4:$\underline{6}$ $\underline{7}$ 8 11 16
5:18	(0,1,3,4,7)	4:$\underline{6}$ 8 12 $\underline{14}$ 17
5:$\underline{19}$	(0,1,3,4,8)	4:$\underline{6}$ 10^2 15^2
5:20	(0,2,3,5,7)	4:$\underline{7}$ 9 10 23 $\underline{25}$
5:21	(0,2,3,5,8)	4:$\underline{7}$ 11 12 $\underline{26}$ 27
5:22	(0,2,4,5,8)	4:$\underline{8}$ 9 15 $\underline{24}$ 27
5:23	(0,1,4,5,7)	4:8 10 $\underline{13}$ 17 20

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
5:24	(0,1,3,6,9)	4:8 11 17 27 [$\underline{29}$]
5:25	(0,2,3,6,8)	4:8 12 16 27 [$\underline{28}$]
5:$\underline{26}$	(0,1,3,5,6)	4:9^2 11^2 $\underline{18}$
5:27	(0,1,3,5,8)	4:9 10 $\underline{19}$ 23 $\underline{26}$
5:28	(0,1,3,5,7)	4:9 12 20 $\underline{22}$ 23
5:29	(0,1,3,6,8)	4:10 11 20 $\underline{25}$ 27
5:30	(0,1,3,7,8)	4:10 12 $\underline{18}$ $\underline{19}$ 20
5:31	(0,1,3,6,7)	4:11 12 16 17 [$\underline{21}$]
5:32	(0,1,4,5,8)	4:$\underline{13}$ $\underline{14}$ 15^2 $\underline{19}$
5:33	(0,1,4,6,9)	4:$\underline{14}$ 16 17 $\underline{26}$ 27
5:$\underline{34}$	(0,1,4,7,8)	4:15^2 17^2 $\underline{18}$
5:35	(0,1,4,6,8)	4:15 16 20 23 $\underline{24}$
5:$\underline{36}$	(0,2,4,6,8)	4:$\underline{22}^2$ $\underline{24}^2$ [$\underline{28}$]
5:$\underline{37}$	(0,2,4,6,9)	4:$\underline{22}$ 23^2 27^2
5:$\underline{38}$	(0,2,4,7,9)	4:23^2 $\underline{25}^2$ $\underline{26}$
6:$\underline{1}$	(0,1,2,3,4,5)	5:$\underline{1}^2$ 2^2 6^2
6:2	(0,1,2,3,4,6)	5:$\underline{1}$ 2 3 $\underline{5}$ 8 17
6:3	(0,1,2,3,4,7)	5:$\underline{1}$ 3 4 7 10 18
6:$\underline{4}$	(0,1,2,3,4,8)	5:$\underline{1}$ 4^2 9^2 $\underline{19}$
6:$\underline{5}$	(0,2,3,4,5,7)	5:2^2 7^2 20^2
6:6	(0,1,2,3,5,6)	5:2 3 6 12 17 $\underline{26}$
6:7	(0,2,3,4,5,8)	5:2 3 9 $\underline{11}$ 21 22
6:8	(0,1,2,3,5,7)	5:2 4 8 14 20 28
6:9	(0,1,2,3,5,8)	5:2 4 10 13 21 27

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
6:10	(0,1,2,3,6,9)	5:3^2 13^2 24^2
6:11	(0,1,2,3,6,7)	5:3 4 12 16 23 31
6:12	(0,1,2,3,6,8)	5:3 4 14 15 25 29
6:13	(0,1,2,3,7,8)	5:4^2 16^2 30^2
6:14	(0,1,3,4,5,7)	5:5 6 7 18 23 28
6:15	(0,2,3,4,6,8)	5:5 8 9 22 25 36
6:16	(0,2,3,4,6,9)	5:5 10^2 24^2 37
6:17	(0,1,2,4,5,6)	5:6^2 8^2 12^2
6:18	(0,1,3,4,5,8)	5:6 7 11 19 27 32
6:19	(0,1,2,4,5,8)	5:6 9 13 18 22 32
6:20	(0,1,2,4,5,7)	5:6 10 14 17 20 23
6:21	(0,1,2,4,6,9)	5:7 8 13 27 33 37
6:22	(0,1,4,5,6,8)	5:7 9 12 30 32 35
6:23	(0,1,2,4,7,9)	5:7 10 14 29 33 38
6:24	(0,1,2,4,6,8)	5:8 9 15 28 35 36
6:25	(0,1,2,4,6,7)	5:8 10 16 26 28 31
6:26	(0,1,2,4,7,8)	5:9 10 16 31 34 35
6:27	(0,1,2,5,6,9)	5:11 12 13 32 33 34
6:28	(0,1,2,5,7,9)	5:11 14^2 35^2 38
6:29	(0,1,2,5,6,7)	5:12^2 14^2 16^2
6:30	(0,1,2,5,6,8)	5:12 13 15 23 25 30
6:31	(0,1,2,5,7,8)	5:13 14 16 29 30 31
6:[32]	(0,1,2,6,7,8)	5:15^2 16^4
6:33	(0,1,3,4,6,7)	5:17^2 18^2 31^2

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
6:34	(0,2,3,5,6,8)	5:17^2 21^2 25^2
6:35	(0,1,3,4,6,9)	5:17 18 21 24^2 33
6:36	(0,1,3,4,6,8)	5:17 19 20 22 29 35
6:37	(0,1,3,4,7,9)	5:18^2 25^2 33^2
6:38	(0,1,3,4,7,8)	5:18 19 23 30 32 34
6:39	(0,2,4,5,7,9)	5:20^2 27^2 38^2
6:40	(0,1,3,5,6,8)	5:20 21 26 27 29 30
6:41	(0,2,3,5,7,9)	5:20 21 28 29 37 38
6:42	(0,1,4,6,7,9)	5:21^2 31^2 33^2
6:43	(0,1,3,5,6,9)	5:22^2 24^2 26 34
6:44	(0,1,3,5,8,9)	5:22 23 27 32 33 35
6:45	(0,1,3,5,7,9)	5:22 25 28 35 36 37
6:46	(0,1,3,6,8,9)	5:23^2 24^2 29^2
6:[47]	(0,1,3,6,7,9)	5:24^2 25^2 31^2
6:48	(0,1,3,5,7,8)	5:27^2 28^2 30^2
6:[49]	(0,1,4,5,8,9)	5:32^6
6:[50]	(0,2,4,6,8,10)	5:36^6
7:1	(0,1,2,3,4,5,6)	6:1^2 2^2 6^2 17
7:2	(0,1,2,3,4,5,7)	6:1 2 3 5 8 14 20
7:3	(0,1,2,3,4,5,8)	6:1 3 4 7 9 18 19
7:4	(0,2,3,4,5,6,8)	6:2^2 7^2 15^2 34
7:5	(0,1,2,3,4,6,7)	6:2 3 6 11 14 25 33
7:6	(0,1,2,3,4,6,9)	6:2 3 9 10 16 21 35
7:7	(0,1,2,3,4,6,8)	6:2 4 8 12 15 24 36

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
7:$\underline{8}$	(0,1,2,3,4,7,9)	6:3^2 12^2 23^2 $\underline{37}$
7:9	(0,1,2,3,4,7,8)	6:3 $\underline{4}$ 11 $\underline{13}$ 22 26 38
7:10	(0,1,3,4,5,6,8)	6:$\underline{5}$ 6 7 18 22 36 40
7:11	(0,2,3,4,5,7,9)	6:$\underline{5}$ 8 9 21 23 $\underline{39}$ 41
7:12	(0,1,2,3,5,6,9)	6:6 7 $\underline{10}$ 19 27 35 $\underline{43}$
7:13	(0,1,2,3,5,6,7)	6:6 8 11 $\underline{17}$ 20 25 29
7:14	(0,1,2,3,5,6,8)	6:6 9 12 20 30 $\underline{34}$ 40
7:15	(0,1,2,3,5,7,9)	6:7 8 12 24 28 41 45
7:16	(0,1,2,3,5,8,9)	6:7 9 11 26 27 $\underline{42}$ 44
7:17	(0,1,2,3,5,7,8)	6:8 9 $\underline{13}$ 25 31 40 $\underline{48}$
7:18	(0,1,2,3,6,7,9)	6:$\underline{10}$ 11 12 30 31 $\underline{46}$ [47]
7:19	(0,1,2,3,6,7,8)	6:11 12 $\underline{13}$ $\underline{29}$ 30 31 [$\underline{32}$]
7:$\underline{20}$	(0,1,3,4,5,7,8)	6:14^2 18^2 38^2 $\underline{48}$
7:21	(0,1,2,4,5,6,8)	6:14 15 $\underline{17}$ 19 22 24 30
7:22	(0,1,3,4,5,7,9)	6:14 15 19 21 $\underline{37}$ 44 45
7:23	(0,2,3,4,6,7,9)	6:14 $\underline{16}$ 20 23 35 41 $\underline{46}$
7:$\underline{24}$	(0,1,2,4,6,8,10)	6:15^2 24^2 45^2 [$\underline{50}$]
7:25	(0,1,3,5,6,7,9)	6:15 $\underline{16}$ 25 26 $\underline{43}$ 45 [47]
7:$\underline{26}$	(0,1,2,4,5,6,9)	6:$\underline{17}$ 18^2 21^2 27^2
7:27	(0,1,2,4,5,8,9)	6:18 19 22 27 38 44 [$\underline{49}$]
7:28	(0,1,2,4,5,7,9)	6:18 20 23 $\underline{28}$ 36 $\underline{39}$ 44
7:29	(0,1,2,4,5,7,8)	6:19 20 26 31 $\underline{33}$ 36 38
7:30	(0,1,2,4,6,8,9)	6:21 22 24 30 44 45 $\underline{48}$
7:31	(0,1,2,4,6,7,9)	6:21 23 25 31 40 41 $\underline{42}$

TABLE XI — continued

N-group	Pcs	(n-1)-group Content
7:32	(0,1,2,4,7,8,9)	6:22 23 26 27 28 29 31
7:33	(0,1,2,4,6,7,8)	6:24^2 25^2 26^2 [32]
7:34	(0,1,2,5,6,8,9)	6:27^2 30^2 37 38^2
7:35	(0,1,3,4,6,7,9)	6:33 34 35^2 37 42 [47]
7:36	(0,1,3,4,6,8,10)	6:34 36^2 41^2 45^2
7:37	(0,1,3,4,6,8,9)	6:35 36 38 40 43 44 46
7:38	(0,1,3,5,6,8,10)	6:39^2 40^2 41^2 48
8:1	(0,1,2,3,4,5,6,7)	7:1^2 2^2 5^2 13^2
8:2	(0,1,2,3,4,5,6,8)	7:1 2 3 4 7 10 14 21
8:3	(0,1,2,3,4,5,6,9)	7:1 3^2 6^2 12^2 26
8:4	(0,2,3,4,5,6,7,9)	7:2^2 6^2 11^2 23^2
8:5	(0,1,2,3,4,5,7,8)	7:2 3 5 9 10 17 20 29
8:6	(0,1,2,3,4,5,7,9)	7:2 3 7 8 11 15 22 28
8:7	(0,1,2,3,4,5,8,9)	7:3^2 9^2 16^2 27^2
8:8	(0,1,3,4,5,6,7,9)	7:4 5 6 12 16 22 25 35
8:9	(0,1,2,3,4,6,8,10)	7:4 7^2 15^2 24^2 36
8:10	(0,1,2,3,4,6,7,9)	7:5 6 8 14 18 23 31 35
8:11	(0,1,2,3,4,6,7,8)	7:5 7 9 13 19 21 29 33
8:12	(0,1,2,3,4,6,8,9)	7:6 7 9 17 18 25 30 37
8:13	(0,1,2,3,4,7,8,9)	7:8 9^2 19^2 32^2 34
8:14	(0,1,3,4,5,6,8,9)	7:10^2 12^2 27^2 37^2
8:15	(0,1,2,4,5,6,7,9)	7:10 11 13 16 26 28 31 32
8:16	(0,1,2,3,5,6,8,10)	7:10 11 14 15 28 30 36 38
8:17	(0,1,2,3,5,7,8,10)	7:11^2 17^2 31^2 38^2

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
8:18	(0,1,2,3,5,6,7,9)	7:12 13 15 18 21 23 25 32
8:19	(0,1,2,3,5,6,8,9)	7:12 14 16 18 29 $\underline{34}$ 35 37
8:$\underline{20}$	(0,1,2,3,5,6,7,8)	7:13^2 14^2 17^2 19^2
8:21	(0,1,2,3,5,7,8,9)	7:15 16 17 19 30 31 32 $\underline{33}$
8:[$\underline{22}$]	(0,1,2,3,6,7,8,9)	7:18^4 19^4
8:23	(0,1,2,4,5,6,8,9)	7:$\underline{20}$ 21 22 $\underline{26}$ 27^2 30 $\underline{34}$
8:$\underline{24}$	(0,1,2,4,5,7,9,10)	7:$\underline{20}$ 23^2 28^2 37^2 $\underline{38}$
8:$\underline{25}$	(0,1,2,4,5,6,8,10)	7:21^2 22^2 $\underline{24}^2$ 30^2
8:26	(0,1,2,4,5,7,8,10)	7:22 23 25 29 31 35 $\underline{36}$ 37
8:[$\underline{27}$]	(0,1,2,4,6,7,8,10)	7:$\underline{24}^2$ 25^4 $\underline{33}^2$
8:$\underline{28}$	(0,1,2,4,5,7,8,9)	7:27^2 28^2 29^2 32^2
8:[$\underline{29}$]	(0,1,3,4,6,7,9,10)	7:35^8
9:$\underline{1}$	(0,1,2,3,4,5,6,7,8)	8:$\underline{1}^2$ 2^2 5^2 11^2 $\underline{20}$
9:2	(0,1,2,3,4,5,6,7,9)	8:$\underline{1}$ 2 $\underline{3}$ $\underline{4}$ 6 8 10 15 18
9:$\underline{3}$	(0,1,2,3,4,5,6,8,10)	8:2^2 6^2 $\underline{9}^2$ 16^2 $\underline{25}$
9:4	(0,1,2,3,4,5,6,8,9)	8:2 $\underline{3}$ 5 $\underline{7}$ 8 12 $\underline{14}$ 19 23
9:5	(0,1,2,3,4,5,7,8,10)	8:$\underline{4}$ 5 6 10 12 16 $\underline{17}$ $\underline{24}$ 26
9:6	(0,1,2,3,4,5,7,8,9)	8:5 6 $\underline{7}$ 11 $\underline{13}$ 15 21 23 $\underline{28}$
9:$\underline{7}$	(0,1,2,3,4,6,7,9,10)	8:8^2 10^2 19^2 26^2 [$\underline{29}$]
9:8	(0,1,2,3,4,6,7,8,10)	8:8 $\underline{9}$ 11 12 18 21 $\underline{25}$ 26 [$\underline{27}$]
9:9	(0,1,2,3,4,6,7,8,9)	8:10 11 12 $\underline{13}$ 18 19 $\underline{20}$ 21 [$\underline{22}$]
9:10	(0,1,2,3,5,6,7,9,10)	8:$\underline{14}$ 15 16 18 19 23 $\underline{24}$ 26 $\underline{28}$
9:$\underline{11}$	(0,1,2,3,5,6,7,8,10)	8:15^2 16^2 $\underline{17}^2$ $\underline{20}$ 21^2
9:[$\underline{12}$]	(0,1,2,4,5,6,8,9,10)	8:23^6 $\underline{25}^3$

TABLE XI – continued

N-group	Pcs	(n-1)-group Content
10:$\underline{1}$	(0,1,2,3,4,5,6,7,8,9)	9:$\underline{1}^2$ 2^2 4^2 6^2 9^2
10:$\underline{2}$	(0,1,2,3,4,5,6,7,8,10)	9:$\underline{1}$ 2^2 $\underline{3}^2$ 5^2 8^2 $\underline{11}$
10:$\underline{3}$	(0,1,2,3,4,5,6,7,9,10)	9:2^2 4^2 5^2 $\underline{7}^2$ 10^2
10:$\underline{4}$	(0,1,2,3,4,5,6,8,9,10)	9:$\underline{3}$ 4^2 6^2 8^2 10^2 [$\underline{12}$]
10:$\underline{5}$	(0,1,2,3,4,5,7,8,9,10)	9:5^2 6^2 9^2 10^2 $\underline{11}^2$
10:[$\underline{6}$]	(0,1,2,3,4,6,7,8,9,10)	9:$\underline{7}^2$ 8^4 9^4
11:$\underline{1}$	(0,1,2,3,4,5,6,7,8,9,10)	10:$\underline{1}^2$ $\underline{2}^2$ $\underline{3}^2$ $\underline{4}^2$ $\underline{5}^2$ [$\underline{6}$]
12:[$\underline{1}$]	(0,1,2,3,4,5,6,7,8,9,10,11)	11:$\underline{1}^{12}$

TABLE XII

Cross-Reference between Forte's Prime Forms and N-groups

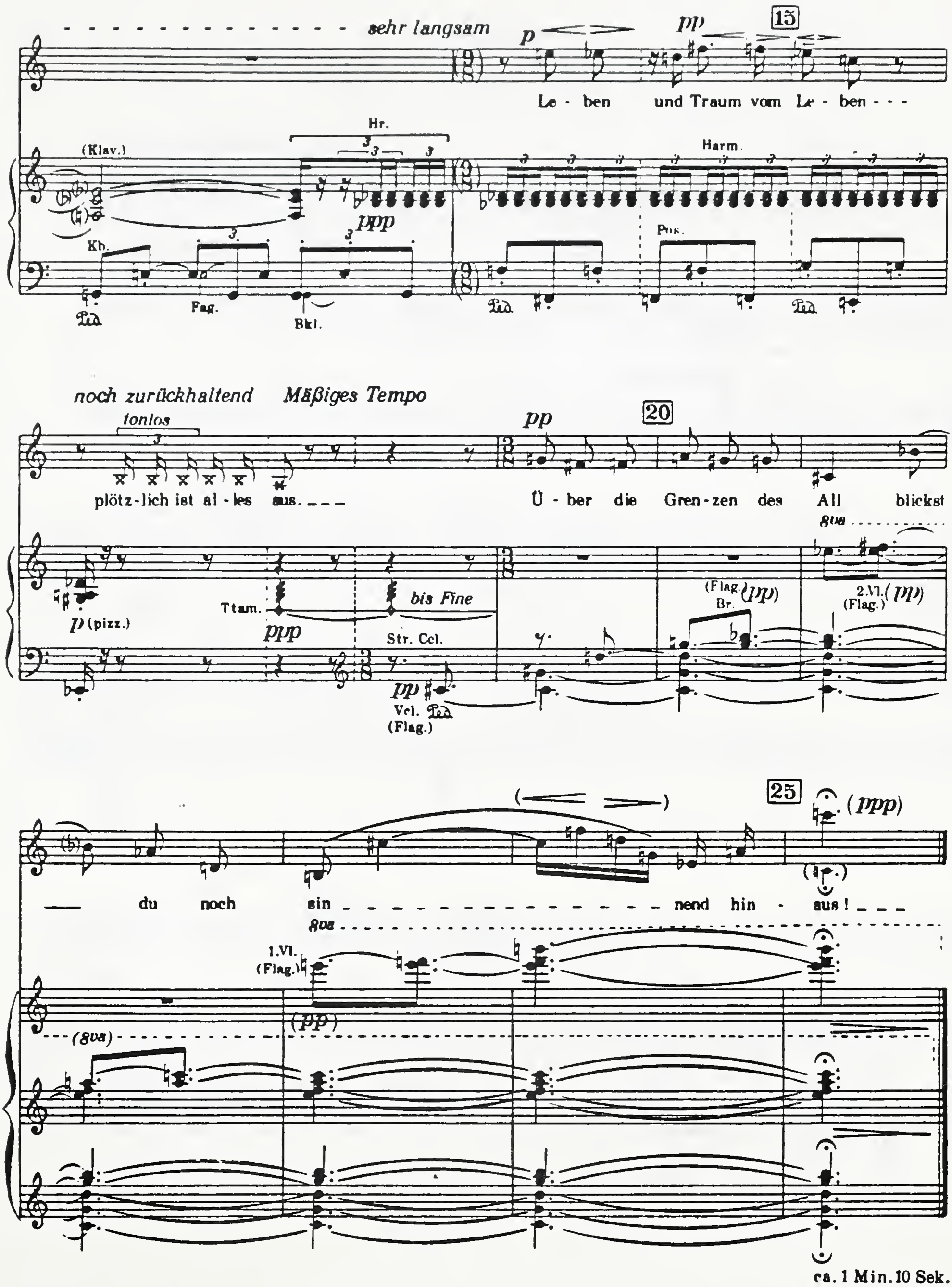
N-group	Prime Form		N-group
3:1	3-1	9-1	9:1
3:2	3-2	9-2	9:2
3:3	3-3	9-3	9:4
3:4	3-4	9-4	9:6
3:5	3-5	9-5	9:9
3:6	3-6	9-6	9:3
3:7	3-7	9-7	9:5
3:8	3-8	9-8	9:8
3:9	3-9	9-9	9:11
3:10	3-10	9-10	9:7
3:11	3-11	9-11	9:10
3:[12]	3-12	9-12	9:[12]
4:1	4-1	8-1	8:1
4:2	4-2	8-2	8:2
4:6	4-3	8-3	8:3
4:3	4-4	8-4	8:5
4:4	4-5	8-5	8:11
4:5	4-6	8-6	8:20
4:13	4-7	8-7	8:7
4:18	4-8	8-8	8:13
4:[21]	4-9	8-9	8:[22]
4:7	4-10	8-10	8:4
4:9	4-11	8-11	8:6
4:8	4-12	8-12	8:8
4:11	4-13	8-13	8:10
4:10	4-14	8-14	8:15
4:16	4-Z15	8-Z15	8:12
4:20	4-16	8-16	8:21
4:14	4-17	8-17	8:14
4:17	4-18	8-18	8:19
4:15	4-19	8-19	8:23
4:19	4-20	8-20	8:28
4:22	4-21	8-21	8:9
4:23	4-22	8-22	8:16
4:25	4-23	8-23	8:17
4:24	4-24	8-24	8:25
4:[28]	4-25	8-25	8:[27]
4:26	4-26	8-26	8:24
4:27	4-27	8-27	8:26
4:[29]	4-28	8-28	8:[29]
4:12	4-Z29	8-Z29	8:18
5:1	5-1	7-1	7:1
5:2	5-2	7-2	7:2
5:6	5-3	7-3	7:3
5:3	5-4	7-4	7:5
5:4	5-5	7-5	7:13
5:12	5-6	7-6	7:9
5:16	5-7	7-7	7:19
5:5	5-8	7-8	7:4
5:8	5-9	7-9	7:7
5:17	5-10	7-10	7:6
5:7	5-11	7-11	7:10
5:26	5-Z12	7-Z12	7:8
5:9	5-13	7-13	7:21
5:14	5-14	7-14	7:17
5:15	5-15	7-15	7:33
5:18	5-16	7-16	7:12
5:19	5-Z17	7-Z17	7:26
5:23	5-Z18	7-Z18	7:16
5:31	5-19	7-19	7:18

TABLE XII — continued

N-group	Prime Form		N-group
5:30	5–20	7–20	7:32
5:32	5–21	7–21	7:27
5:34	5–22	7–22	7:34
5:20	5–23	7–23	7:11
5:28	5–24	7–24	7:15
5:21	5–25	7–25	7:23
5:22	5–26	7–26	7:22
5:27	5–27	7–27	7:28
5:25	5–28	7–28	7:25
5:29	5–29	7–29	7:31
5:35	5–30	7–30	7:30
5:24	5–31	7–31	7:35
5:33	5–32	7–32	7:37
5:36	5–33	7–33	7:24
5:37	5–34	7–34	7:36
5:38	5–35	7–35	7:38
5:10	5–Z36	7–Z36	7:14
5:11	5–Z37	7–Z37	7:20
5:13	5–Z38	7–Z38	7:29
6:1	6–1		
6:2	6–2		
6:6	6–Z3	6–Z36	6:3
6:17	6–Z4	6–Z37	6:4
6:11	6–5		
6:29	6–Z6	6–Z38	6:13
6:[32]	6–7		
6:5	6–8		

N-group	Prime Form		N-group
6:8	6–9		
6:14	6–Z10	6–Z39	6:7
6:20	6–Z11	6–Z40	6:9
6:25	6–Z12	6–Z41	6:12
6:33	6–Z13	6–Z42	6:10
6:18	6–14		
6:19	6–15		
6:22	6–16		
6:26	6–Z17	6–Z43	6:30
6:31	6–18		
6:38	6–Z19	6–Z44	6:27
6:[49]	6–20		
6:15	6–21		
6:24	6–22		
6:34	6–Z23	6–Z45	6:16
6:36	6–Z24	6–Z46	6:21
6:40	6–Z25	6–Z47	6:23
6:48	6–Z26	6–Z48	6:28
6:35	6–27		
6:43	6:Z28	6–Z49	6:37
6:46	6–Z29	6–Z50	6:42
6:[47]	6–30		
6:44	6–31		
6:39	6–32		
6:41	6–33		
6:45	6–34		
6:[50]	6–35		

APPENDIX 2: ALBAN BERG: OPUS 4/3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APPENDIX 3

ANTON WEBERN: OPUS 7/3

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